

EE13

Updating Selected Laboratories

for Engineering Experimentation Course at Worcester Polytechnic Institute

A Major Qualifying Project Report

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Abstract

This project aimed at updating two existing laboratories in Engineering Experiment course at Worcester Polytechnic Institute: Strain and Pressure Measurement Laboratory and Vibration Measurement Laboratory. Without major alternation to the experiment designs, the author examined laboratory equipment, software and instruction materials for both laboratories.

Two alternative signal conditioners are recommended as replacements to the current system; software is updated to improve usability; and comprehensive laboratory instructions are developed based on existing materials. The updated materials are suitable for future use at WPI and other universities.

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This project builds upon existing laboratories developed faculty at WPI Mechanical Engineering Department. I also want to acknowledge the wonderful work they have done with this course.

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1. Introduction

ABET accreditation criteria requires undergraduate engineering programs to prepare students with “an ability to design and conduct experiments, as well as to analyze and interpret data”, and mechanical engineering curriculums to “require students to apply principles of engineering, basic science and mathematics”, and “prepare students to work professionally in both thermal and mechanical systems.” (ABET, 2012)

The Engineering Experimentation course at WPI is designed to “develop analytical and experimental skills in modern engineering measurement methods, based on electronic instrumentation and computer-based data acquisition systems” (WPI, 2012). The lectures address topics in engineering analysis and design, and the principles of instrumentation. (WPI, 2012)

It is the common understanding that the current laboratories in the WPI Engineering Experimentation course can benefit from further efforts of improvement. Two existing laboratory experiments from the Engineering Experimentation curriculum are selected as targets of improvement: the Pressure Measurement Laboratory and the Vibration Measurement Laboratory.

The Pressure Measurement Laboratory performs characterization of internal pressure in soda cans by measurements of strain, and conducts uncertainty analysis of characterized internal pressures. (Furlong, Lab: Strain and Pressure Measurement, 2013) The Vibration Measurement Laboratory determines the vibration amplitude, velocity, and acceleration, determines natural frequencies and damping characteristics, and estimates elastic modulus of the cantilever used. The results are then compared with analytical and/or computational models, uncertainty analysis is then performed. (Furlong, Lab: Vibration Measurement, 2013)

This project aims at improving the laboratories by updating the instructive materials and experiment equipment. The experiment designs will remain unaltered, suggestions for expanding the experiment designs are included for future reference. The aspects for improvement include:

- Educational Value.

Provided instructive materials and equipment should assist the students to meet the defined objectives of the laboratories, and gain critical understanding of the physical phenomenon, experimentation concepts, instrumentation principles, and data analysis

techniques involved. The equipment should enable acceptable level of accuracy and resolution.

- Student Experience.

The instructive materials should prepare the students with relevant theoretical background, give clear and organized instructions that are easy to follow, and provide challenging questions and assignments. The equipment should be safe and easy to operate, and it should enable hands-on experience to deepen students' understanding of the equipment's operating principle. Basic troubleshooting instructions should be provided to the students and instructors to assist problem solving.

- Cost-Effectiveness

Equipment for the laboratories should be low cost and adaptable for other purposes of use. The perishable supplies should provide sufficient learning experience before being consumed.

This project closely evaluates the existing laboratories and practices at other institutes regarding similar experiments, and identifies goals for improvement according to the evaluation. The identified issues are then addressed, and the updated laboratory is reevaluated after solutions are implemented. Afterwards, opportunities to expand the experiment designs are explored.

In this project, an alternative strain gauge amplifier is selected from in-lab preliminary evaluation of four suitable products; updated instructive material and sample reports are generated for future Engineering Experimentation sections at WPI; and a literature research is conducted to identify opportunities of expansion upon the current experimentation designs.

This report documented the process of this project in detail. The background section, includes descriptions of the experiment procedures, sample results, an overview of practices at other institutes, and identified areas for improvement of the existing laboratories. The methodology section includes the method used to evaluate and compare the existing and potential alternative strain gauge amplifiers, and the method used to evaluate the measurements acquired from the selected alternative amplifier.

2. Background

2.1 Contents and Implementation of the Course

The Engineering Experimentation course at WPI is a junior-level compulsory course for the Mechanical Engineering program; the coursework is equivalent to 3 credits. Before taking this course, students are recommended to finish the compulsory fundamental courses in ordinary differential equations, thermal-fluid, mechanics, and material science.

The topics addressed in this course include: review of engineering fundamentals, discussions of standards, measurement and sensing devices, experiment planning, data acquisition, analysis of experimental data, and report writing, etc. The laboratory periods exposes students to modern devices in experiments, which involves both mechanical and thermal systems and instrumentation in mechanical engineering. The students are trained to use modern measurement and data acquisition systems, to process data with computer software, and to produce formal laboratory reports.

The course provides two one-hour lectures and two three-hour lab periods every week throughout a WPI's seven-week term. Five laboratories with various focus and difficulties are included.

2.2 Comparable Curriculums in Other Mechanical Engineering Programs

As essential parts of Mechanical Engineering skillset, topics from engineering experimentation methods and techniques are often covered in undergraduate required curriculums, and many programs provide electives for students to utilize and to further strengthen skills in this area. The lectures are often accompanied by weekly laboratories that focus on important topics mentioned in lectures; term projects are sometimes required.

The author conducted a primary curriculum study performed on around 70 top-ranked undergraduate Mechanical Engineering programs according to the information from university catalogs and course content websites. This study helped to identify the topics frequently taught at state-of-art education institutes. Application of probability and statistics, uncertainty analysis, calibration, resolution, precision, basic measurement techniques, basic signal conditioning, and technical communication are addressed by almost all programs investigated. However, these topics are often scattered in several courses taught at different times.

Familiarity of computerized data acquisition system is cultivated in required courses in the majority of programs investigated, and a significant amount of programs teaches the use of traditional tools such as oscilloscope and function generator. Other frequently taught topics include frequency response and Fourier transformation, experiments in fluid mechanics, and material testing. In the programs where control theory is a required content, experimentation laboratories often include a section in control as well.

However, there are at least seven programs among the investigated ones that do not dedicate coursework to address experimentation methodology and techniques, or introduces these contents in a half course laboratory focused in other subjects such as material property.

2.3 Current Design and Implementation of the Laboratories

The laboratories cover measurements of resistance, calibration of pressure sensors, measurement of pressure with strain gauges, measurement of vibration with strain gauges, and calibration of thermistors.

Data acquisition system is used in all the laboratories, where LabVIEW programs are used to facilitate data acquisition and processing. Mathematical programs are used in data analysis. A formal lab report is required for each laboratory.

The lengths of the laboratories vary from one three-hour lab period to four periods. During the lab periods, the instructor explains the procedures and important issues, and helps students with trouble shooting when they are performing the experiment.

The Strain and Pressure Measurement laboratory teaches pressure characterization of thin-wall vessel, uncertainty analysis of the result is also required. Combined with the signal conditioning system and analog/digital converter, strain gauges are utilized to detect the change in surface strain when the soda cans are opened; the strains are transferred to sensitive readings in voltage with a Wheatstone bridge, and then used to determine the pressure difference after it is amplified with a pre-amplifier.

The Vibration Measurement Laboratory focuses on dynamic measurement of a free-vibrating cantilever beam, the measured characteristics are then used to estimate the elastic modulus of the beam; uncertainty analysis of the results is also required. Same set-up is used to

acquire data in strains as the Strain and Pressure Measurement Laboratory, including strain gauges, a Wheatstone bridge, a signal conditioning system, and an analog/digital converter.

2.2.2 Opportunities for Improvements and Expansions

Laboratories that measures pressure and vibration with strain gauges are frequently taught at colleges. A review of the similar laboratories developed by other organizations provides insights for the opportunities for improvements and expansions of the current laboratories at WPI. More areas for improvements are suggested from a close examination of the current laboratories.

2.2.2.1 Design of Strain and Pressure Measurement Laboratory

The current set-up provides reliable measurements with an impressive micro-strain level resolution, which is enabled by the 16-bit A/D converter and the signal conditioner. However, the current laboratory does not fully utilize the set-up to reinforce the students' appreciation of resolution and the benefits of signal conditioning in the laboratories. The students can benefit from additional activities to measure the resolution and noise under different gain settings and the A/D converter's range settings.

The current laboratory focuses on demonstrating pressure measurement with strain gauges. The strain gauges are installed on soda cans, and once the cans are opened, the gauges cannot be reused. Adding a few more activities to the existing laboratory could help utilize the perishable materials more sufficiently. For example, an experiment conducted at Purdue University College of Technology at New Albany proved that turning the can end to end twice after shaking the can for 5 seconds will drastically reduce the measured strain in soda can experiment. (Dues, 2006) The results from this experiment showed that the strain measured after turning the can end to end twice is 90% lower than the strain measured directly after shaking the can for 5 seconds. (Dues, 2006)

It is not uncommon for undergraduate laboratories to take advantage of a "class data sheet" to increase the size of data set, which often provides more opportunity to practice the statistical data analysis, and to draw significant conclusions to the phenomenon investigated. In the case of this specific laboratory, multiple sources of errors exist in the strain gauge installation process, such as alignment, surface condition of the adherence, wiring, etc. The property measured, internal pressure of a soda can, can be easily affected by the person conducting the measurement, as the solubility of carbon dioxide in the beverage can be affected as local

temperature and pressure vary, which unsettles the equilibrium of fluids and carbon dioxide in the can. For these stated sources of variability, the students can gain more insights of the experiment from a relatively large amount of measurements. Therefore, use of a “class data sheet” to gather experiment results can be beneficial. In addition, due to the different carbonation processes in the beverage products, beverage cans of one product should be provided to the whole class to acquire more meaningful data.

Strain gauge rosettes, or a set of single strain gauges arranged in different directions, can characterize the principle stresses of a surface. Strain measurements in multiple axes can provide valuable learning experience to students. A student laboratory used at Middle East University uses a three element rectangular rosette with an unknown angle α to the axis of a thin-wall vessel to measure the internal pressure of the vessel. (Middle East Technical University) The material's Poisson's ratio, the unknown angle α , the direction and magnitude of principle stresses, principle strains are also determined as a part of the laboratory. More in-depth engineering calculations are utilized in the mentioned laboratory, and the introduction of strain rosettes adds breadth to the basic design of laboratory.

2.2.2.2 Design of Vibration Measurement Laboratory

Since the vibration measurement laboratory shares the same set up with the strain and pressure measurement laboratory, investigation of the resolution and noise under different gains and the A/D converter's range settings can be conducted in this laboratory as well.

Calibration of the strain measurement system can be taught with the current setup of this laboratory. A laboratory taught at Northern Illinois University requires students to determine the strain gauges' gage factors by applying known weights, which directly leads to known strains. (Mostic, 1990) A laboratory taught at University of Massachusetts Lowell teaches students to determine the systems' sensitivity by applying known deflection, so that the output voltage can be associated with measured strain. (Mostic, 1990) The laboratory developed by Vernier Software and Technology challenges the students to use the set-up as a penny-counter, which will accurately count the number of pennies placed on the tip of the cantilever. (Mostic, 1990)

This laboratory teaches students to determine elastic modulus of the material by deriving it through measured frequency, yet the determination of “effective length” for vibration of the beam, which is a parameter in the calculation, is somewhat ambiguous in this laboratory. The

error in finding the “effective length” adds uncertainty to the elastic modulus estimation. The boundary condition created by the fixture made of a c-clamp and the lab bench makes the set-up slightly deviates from the simple cantilever model. The issue can be alleviated by simply adding a piece of aluminum plate between the clamp and the beam. A special fixture tool with a rectangle shape that rigidly clamps the beam can provide the ideal boundary condition. On the other hand, a conventional method of determining elastic modulus can be used in conjunction to the existing method for comparison: apply a series of known load to the tip of the beam while measuring strain, so the stress-strain plot can be created for extracting the elastic modulus.

The current laboratory uses a quarter-bridge Wheatstone set-up. The laboratories can be extended by introducing half-bridge and full bridge set-up for uniaxial bending strain measurement. In addition to providing temperature compensation and higher sensitivity, the different set-ups enable compensation for the effect of transverse strain, and/or for the effect of lead wire resistance. (Acromag)

Multi-axial strain measurement with a cantilever opens up opportunities for enriching the current vibration laboratory and developing new laboratories. Besides principle stress and strain under bending, which can be directly derived from strained measured in different directions, Poisson’s ratio and stress concentration factor are among the characteristics the set-up can measure.

Poisson’s ratio of the material can be measured with two strain gauges mounted along longitudinal and lateral axis at corresponding locations on opposite sides of the beam. In a laboratory used at Arizona State University, a range of known stresses is applied to acquire the corresponding longitudinal and lateral strain when the beam reaches stable state. The dataset is examined for outliers and then used to calculate an average Poisson’s ratio. (Poisson's Ratio, 2003)

Stress concentration factor of a specific geometric discontinuity, such as a hole on a cantilever, can be measured with a group of very small strain gauges with fine grids. The characterization can be performed when the beam is loaded with point bending and/or tension with different methods. The laboratory taught at New Mexico Tech fitted the measured data into a stress distribution function to find the constants, which contribute to the calculation of the concentration factor. (Ruff) Other data analysis methods for laboratories range from deliberate

Finite Element simulation (Kargar, S; Bardot, D.M. ., 2010) to simplified method of drawing a smooth curve to connect the data points on a plot with distance from the hole against the local strain (Stress and Strain Concentration, 2003).

When a set of strain gauges are installed at different positions on the longitudinal axis of a cantilever beam, and a known bending load applied at the free end of the beam, a series of beam characteristics can be determined, and the system can be calibrated to measure unknown loads. A stress analysis laboratory taught at Middle East Technical University with such design determines the beam's flexural rigidity, height, location of the beam and location of the applied load, and enables the system to measure loads. (Middle East Technical University)

The current laboratory derives natural frequency of the beam by analyzing the vibration data during free vibration. Another alternative method to determine natural frequency is to find the resonance profile of the beam, or the amplitude response at each frequency under forced vibration of constant amplitude input. This method is taught in certain curriculums, for example, in the Aerospace laboratory course at University of Toronto. (Emami)

2.2.2.3 Equipment

While some improvements can be implemented without major change in the equipment list, the existing resources do not support measurement of strain in multiple axes. The employed signal conditioning unit, Vishay 2310 Signal Conditioning Amplifier System, is single-channeled and relatively costly to acquire. An economical yet effective alternative signal conditioner should be acquired to enable this upgrade, which will be discussed in section 3.1 and section 4.1 of this report.

2.2.2.4 Laboratory Instructive Materials

The instructive materials should guide the students to prepare for the laboratory, conduct the experiment and complete the lab report. Background knowledge of the key elements should be provided to ensure sufficient appreciation of the topics; the experiment steps should be described in details; and the discussion questions should help reinforce the knowledge and skills acquired.

To help students further appreciate the Wheatstone bridge set-up with strain gauges, the lab assignment should include calculations of current draw and power consumption of the circuit.

2.2.2.5 Software

The course requires no previous experience of programming in LabVIEW, therefore, the instructive materials need to guide the students to construct simple functional programs to facilitate the experiment. More advanced LabVIEW programs can be provided to the students to use in the laboratories. Some of the suitable features to add to the basic programs are: filters, data processing, and data reporting.

3. Methodology

3.1 Selection of Alternative Signal Conditioners

3.1.1 Selection of Potential Alternative Signal Conditioners for In-Lab Testing

The two target laboratories, strain and pressure measurement laboratory, and vibration measurement laboratory, use the same set up for strain measurement. The hardware set-up includes a signal conditioner unit that supplies excitation voltage and amplifies output signal from the Wheatstone bridge. The amplified signal is converted to digital signal by the A/D converter, and then processed by software installed on a computer. Currently, NI6229, a 16 bit DAQ instrument is used for the conversion, and LabVIEW program is used for data processing. Figure 1 shows the system schematic of the strain measurement set-up.

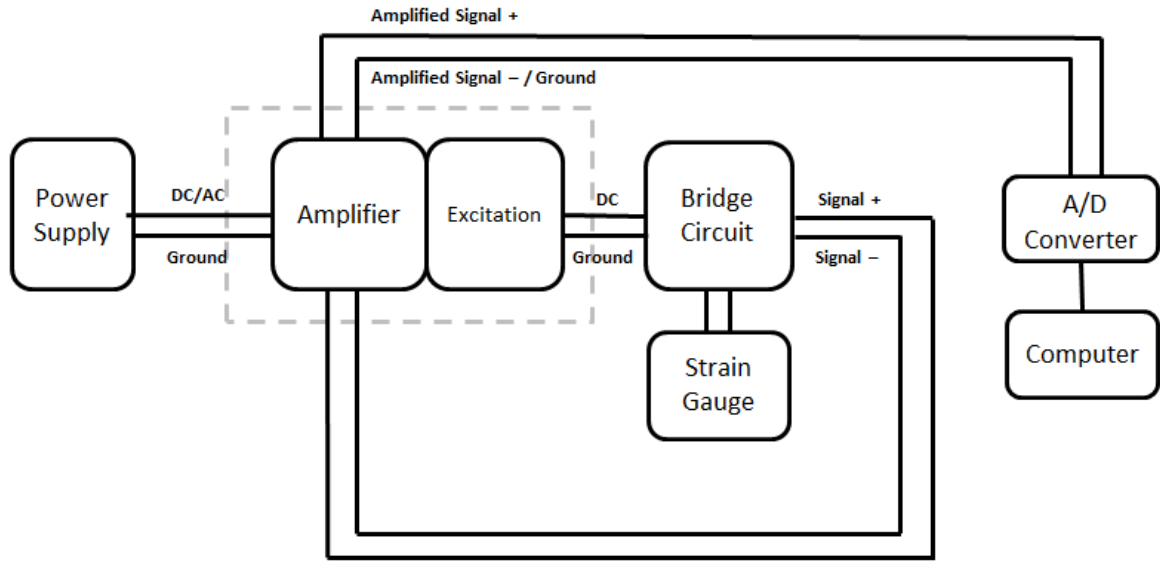


Figure 1 System Schematic of Strain Measurement Set-up

The resolution of the experiments is determined by the signal conditioner gain setting, A/D converter's resolution, and its range. The range can be configured LabVIEW DAQ assistant module. Since the resolution of an A/D converter of N bit is given by expression:

$$Resolution = \frac{Range}{2^N}$$

where N , the effective bit is 1.5 bit less than total bit.

In the case of the current set-up, the range can be set to 5V. Thus, the resolution is 0.22mV. Amplifier gain determines the transfer function between signal voltage and represented strain. When gain is 200, the resolution represents 0.23 microstrain.

In order to enable successful implementation of the laboratories, suitable signal conditioners should satisfy a series of requirements. Both laboratories use strain gauges with 120 Ω resistance, the signal conditioner should be able to provide quiet, accurate and stable readings with selected strain gauges. The laboratories currently use a gain setting close to 200; the signal conditioner should be able to offer a comparable gain. The vibration measurement laboratory requires the signal conditioner to have a frequency response of over 1kHz. Gain indication and bridge balancing are preferred, optional internal Wheatstone bridge completion circuit is another preferred feature. The cost for each unit of signal conditioner should be relatively low, so that experiments involving multiple channels of strain measurement could be developed in the future.

The currently deployed signal conditioner is Vishay 2310 Signal Conditioning Amplifier System (Vishay 2310). Extensive search of available commercial products was conducted to select the potential alternative systems that satisfy most criteria listed above.

3.1.2 In-Lab Output Testing

In-lab testing of the signal conditioners' output is conducted. A quarter bridge completion board is used for testing of all signal conditioners except Vishay 2310, which contains an internal quarter bridge completion circuit. The variable strain gauge is installed on an aluminum cantilever beam with one foot of gauge 26 wires. For Omega, the gain was adjusted so that 1V signal output corresponds to 800 microstrain. Table 1 shows the excitation and gain selected for each signal conditioner for this testing. The signal conditioners require time to "warm up", so that optimum performance can be achieved. Table 1 shows the warm-up time used for each signal conditioner during testing.

	Vishay 2310	Omega DMD-465WB	Honeywell UV-10	DATAQ DI5B38-04	Tacuna
Excitation	10V	10V	5V	10V	5V
Gain	191	239*	200	333.3	220
Warm-up Time	0	45 min	10 min	10 min	5 min

Table 1 Parameters for Testing

Strain load can be simulated when a shunt resistor is connected to the Wheatstone bridge in parallel to the active gauge. For the first output testing, three different shunt resistors are used to simulate three different strains for each signal conditioner. For each shunt resistor, five readings are taken with each signal conditioner. The three shunt resistors used in this testing have the resistance of 12.88kΩ, 35.92kΩ, and 45.05kΩ, simulating strains of 4406 micro strain, 1751 micro strain, and 1268 micro strain respectively.

The shunt resistor is connected to the circuit before the recording starts. It is disconnected from the circuit approximately five seconds after the recording starts. The recording continues for 10 to 15 seconds until the readings stabilize for at least five seconds. A voltage reading is acquired by calculating the absolute difference between the average voltage before disconnecting the shunt resistor, and the average voltage after the signal stabilizes. The strain reading is given by expression:

$$\varepsilon = \frac{4V_{in}}{Gain \times V_E \times F}$$

where V_{in} is the voltage reading, $Gain$ is the amplifier gain, V_E is the excitation voltage, and F is the excitation voltage.

The strain readings are evaluated for noise, accuracy and precision. The results of these three parameters are compared for the four tested systems to eliminate systems with undesirable performance.

The *Error* is the percentage difference between measured strain and simulated strain. The *Maximum Error* is the largest observed error in the five samples taken. The *Average Error* is given by:

$$\bar{\delta} = \sqrt{\frac{\delta_1^2 + \dots + \delta_{15}^2}{15}}$$

where δ_i is the *Error* of each reading.

For each reading, the standard deviation of the signals after applying simulated load is calculated; this is equivalent to RMS noise. Peak-to-peak noise amplitude is within 8 times RMS value 98% of the time.

Note that the sources of measured output noise not only include the components in the signal conditioner, but also include other hardware in the set-up. The set-up is designed to simulate real conditions in the laboratories.

A second output testing is conducted with actual strain. With the clamped cantilever, a known weight is attached to the free end of the beam, simulating strains between 200 micro strain and 300 micro strain. Accuracy of 3 signal conditioners is tested by this method.

The test results for all four systems are reviewed and compared to eliminate systems with less desirable performances.

3.1.3 Comparison of Set-up Procedure

The laboratories require systems that are easy and safe for students to connect and balance. Gain setting and gain indication mechanisms are necessary. Among the signal conditioners with acceptable performance (determined by in-lab output testing), the connection layout, balancing procedure and gain setting procedure are compared to eliminate the ones unsuitable for use in these laboratories.

Thus, the remaining systems should have both acceptable performance and suitable set-up procedures to be used in selected laboratories.

3.1.4 Test Run of the Laboratories with Selected Signal Conditioners

Test run of the laboratories are performed with selected signal conditioners for further validation. Sample laboratory reports are generated for reference.

3.2 Updating Software and Laboratory Instruction Materials

The current VIs is examined. Improvements are made to the program for usability, without any major alternation of the program design. Detailed step-by-step tutorial for constructing these VIs are written.

In the updated instructions, the key background topics are identified for each laboratory, brief overview of the topics and derivations of key equations are provided. Optional activities are developed for each laboratory based on the selected improvement opportunities for laboratory designs, which are identified in the Background section. The optional activities are described in attached documents to the instructions. The set-up and use procedures for the selected alternative signal conditioner are included in the instructions. The detailed descriptions for the experiment procedures are written based on the current instructions. Current materials include precise slides and the instructor needs to explain the steps in person.

4. Results

This project recommended two alternative signal conditioners for use in Strain and Pressure Measurement Laboratory and Vibration Measurement Laboratory, and updated the LabVIEW programs and instruction materials for both laboratories.

4.1 In-Lab Evaluation of Alternative Signal Conditioners

Extensive search for available commercial products, in-lab output testing for selected potential products, and comparison of set-up procedures are conducted to generate recommendations for alternative signal conditioners. Test runs of specified labs are conducted to verify the conclusions.

4.1.1 Basic Information

The basic parameters of Vishay 2310 are compared with potential alternative signal conditioners in Table 2. The selected products include Omega DMD-465WB Bridge Sensor AC Powered Signal Conditioner (Omega DMD-465WB), Honeywell UV-10 In-line Amplifier (Honeywell UV-10), Tacuna Systems Strain Gauge or Load Cell Amplifier/Conditioner Interface Manual (Tacuna), and DATAQ Instruments DI5B38-04 Strain Gage Input Modules (DATAQ DI5B38-04). All five systems are single-channeled.

As shown in Table 2, all four alternative systems satisfy the basic requirements regarding gain and frequency response. Honeywell and Tacuna meet the preferred requirements of enabling bridge balance and gain indication.

	Vishay 2310	Omega DMD-465WB	Honeywell UV-10	DATAQ DI5B38-04¹	Tacuna
Supply Voltage	115VAC	115VAC	18 - 32 VDC	5VDC	6-16VDC
Excitation Voltage	0.5-15 VDC, 12 settings	4 to 15VDC Manual adjust	3, 5 and 10 V, Manual adjust	10VDC	5VDC
Current Draw	Max 100 mA	Max 120mA	Max 70mA	70-170mA	Max 100 mA
Bridge Balance	Automatic	N/A	Manual	N/A	Manual
Gain	1-11000 continuous variable	40–250, manual adjust potentiometer	37.5-1000, 12 switch settings	333.3	110-11000, seven switch settings; Precision programmable gain
Gain Indication	Direct reading	No indication	Shown by switches on circuit board	No indication	Shown by switches on circuit board
Output Noise	5 μ V RMS	0.2 mV RMS (when Gain=100)	55 μ V RMS	2mV RMS	Data unavailable
Frequency Response	60kHz	2kHz	5kHz	10kHz	Factory Adjustable
Size	8.75"H×1.706"W ×15.9" D	3.75"L× 2.0"W ×2.87"W	3.75"L×2.5"W ×2.1"H	2.5" L× 2.5"W ×1.0"H	3.3" L× 1.3"W × 1.0"H
Cost	\$2,200	\$363	\$350	\$225	\$117

Table 2 Comparison of Signal Conditioners' Basic Parameters

(Vishay Precision Group)

(Omega)

(Honeywell)

(Dataq)

(Tacuna Systems)

4.1.2 Output Testing

With the method described in section 3.1.2 of this report, the noise, accuracy, and precision of measurements are evaluated. Table 3 shows the analysis results when the strains are simulated with shunt resistors.

	Vishay 2310	Omega DMD-465WB	Honeywell UV-10	DATAQ DI5B38-04	Tacuna
Noise					
Output Noise (mV RMS)	0.24	2.46	2.14	4.99	0.18
Output Noise (microstrain RMS)	0.24	3.94	4.09	11.44	0.31
Output Noise (mV Peak-to-Peak)	1.93	19.71	16.8	39.95	1.44
Output Noise (microstrain Peak-to-Peak)	1.93	31.49	32.7	91.54	2.48
Accuracy					
Maximum Error	2.6%	9.3%	2.9%	12.5%	3.5%
Average Error	1.7%	7.1%	1.7%	9.7%	2.1%
Precision					
Standard Deviation of Readings* (mV)	0.65	0.36	0.54	4.55	3.90
Standard Deviation of Readings* (microstrain)	0.65	0.57	1.04	10.43	6.77

Table 3 Shunt Resistor Testing Result for Signal Conditioners

While the other tested signal conditioners responses to the change in simulated load instantaneously, DATAQ occasionally shows response time as long as 3 to 5 seconds. Figure 2 shows the amplified signal output of DATAQ when the measurement starts at second 0, and simulated load is removed at second 10.

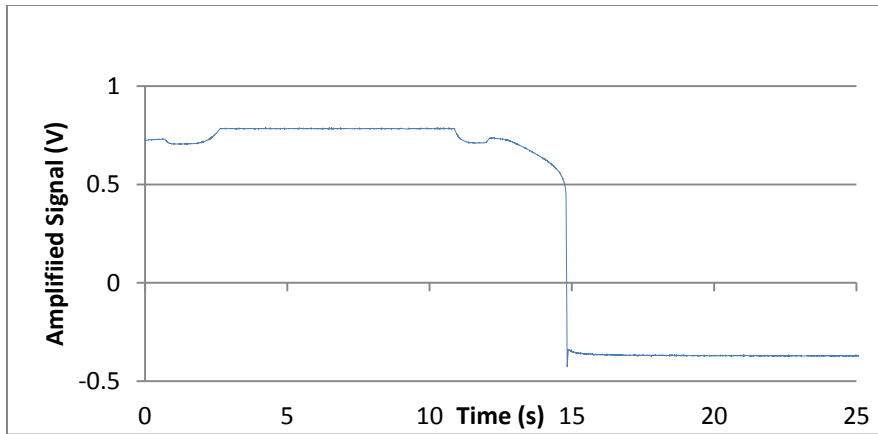


Figure 2 DATAQ's Signal Response to Shunt Resistor

With the method described in section 3.1.2 of this report, the noise, accuracy, and are evaluated with actual strain of known values. Table 4 and Table 5 show the analysis results.

Applied Load (micro strain)	Omega DMD-465WB	Honeywell UV-10	Tacuna
266.8	265.7	258.9	259.6
240.5	234.9	225.3	234
213.7	209.8	207.9	208.7
Error	2%	4%	2.5%

Table 4 Accuracy Test Result

	Vishay 2310	Omega DMD-465WB	Honeywell UV-10	DATAQ DI5B38-04	Tacuna
Output Noise (mV-RMS)	0.30	0.55	0.72	1.25	0.14
Output Noise (microstrain-RMS)	0.30	0.69	0.75	2.19	0.16
Output Noise (mV-Peak to Peak)	2.38	4.43	5.77	10.05	1.10
Output Noise (microstrain- Peak to Peak)	2.38	5.53	6.05	17.54	1.27

Table 5 Output Noise of Signal Conditioners

As shown in the test results, the current system, Vishay 2310, outperforms all other system in all measured aspects. It is, however, significantly higher cost than all other systems considered. The measured performance metrics for the four considered amplifier are ranked from 1 to 4, 1 being the best performance and 4 being the worst. As shown in Table 6, DATAQ DI5B38 shows the lowest performance compared to other systems, and therefore should be eliminated from the selection.

	Omega DMD-465WB	Honeywell UV-10	DATAQ DI5B38-04	Tacuna
Noise	2	3	4	1
Accuracy	3	1	4	2
Precision	1	2	4	3

Table 6 Comparison Metrics for the Signal Conditioners

4.1.3 Set-up Procedure

The set-up procedures for Tacuna, Honeywell and Omega are described and compared. Discrete gain setting mechanism and gain indication are desirable features, and devices with easier connecting procedures are preferred.

4.1.3.1 Set-up Procedure for Tacuna

1. Connection

Connect the wires as indicated in Figure 3.

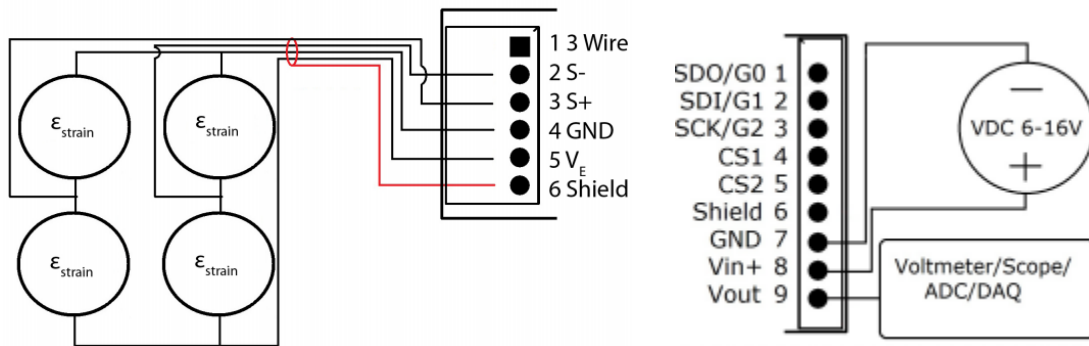


Figure 3 Connections for Tacuna Systems Strain Gauge or Load Cell Amplifier/Conditioner Interface Manual

2. Gain Setting

To get a gain of 220, make sure the switches (location shown in Figure 4) are set as indicated in Table 7.

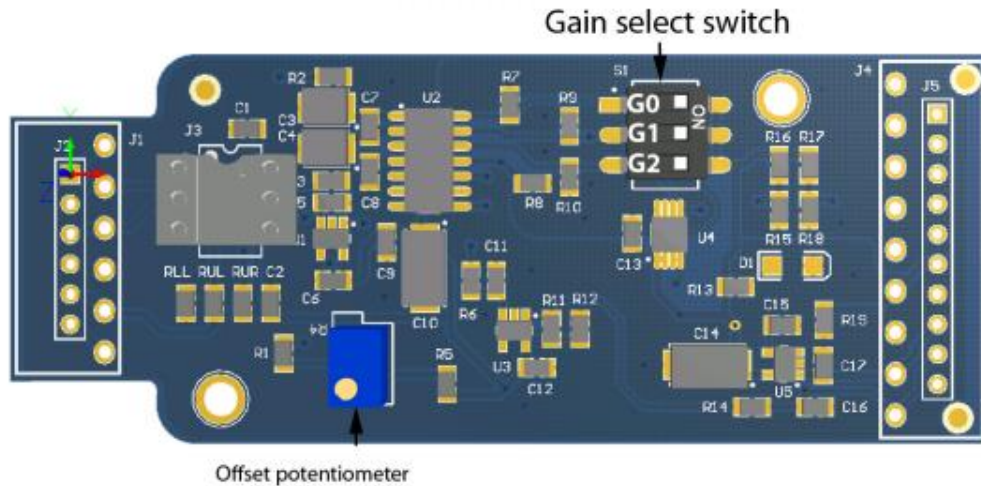


Figure 4 Location of Gain select switch and offset potentiometer

G0	G1	G2
ON	OFF	OFF

Table 7 Switch settings for Tacuna for 220 Gain

3. Bridge Balance

Use the offset potentiometer to adjust the output voltage to 2.5V, which is half of the output range.

It is required to open the enclosure to adjust the gain switches but not the offset potentiometer. The wire connections are located outside of the enclosure.

4.1.3.2 Set-up Procedure for Honeywell UV-10

1. Connection

Connect the wires as indicated in Figure 5.

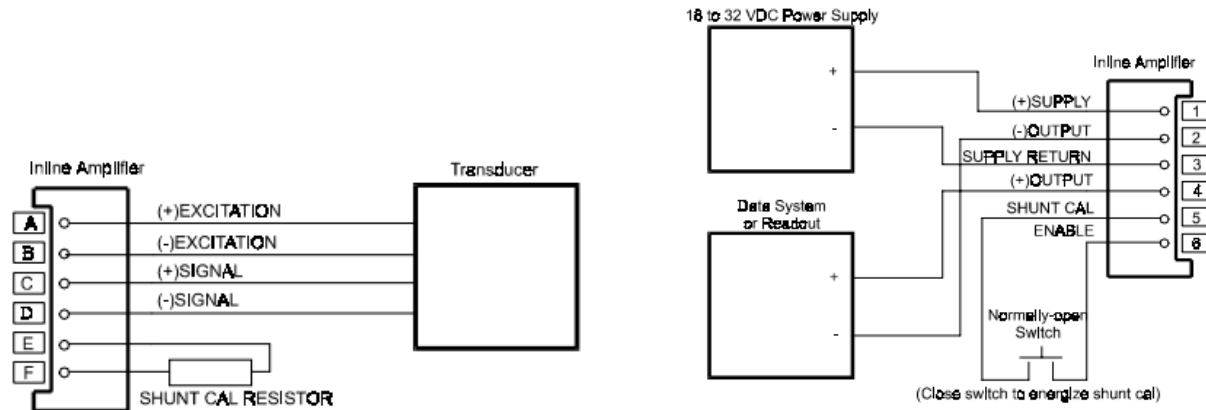


Figure 5 Connections for Honeywell UV-10 In-line Amplifier

2. Gain setting

While excitation jumper is set to 5V, in order to get a gain of 200, make sure switch 3 and switch 6 are on and other switches are off.

3. Calibration

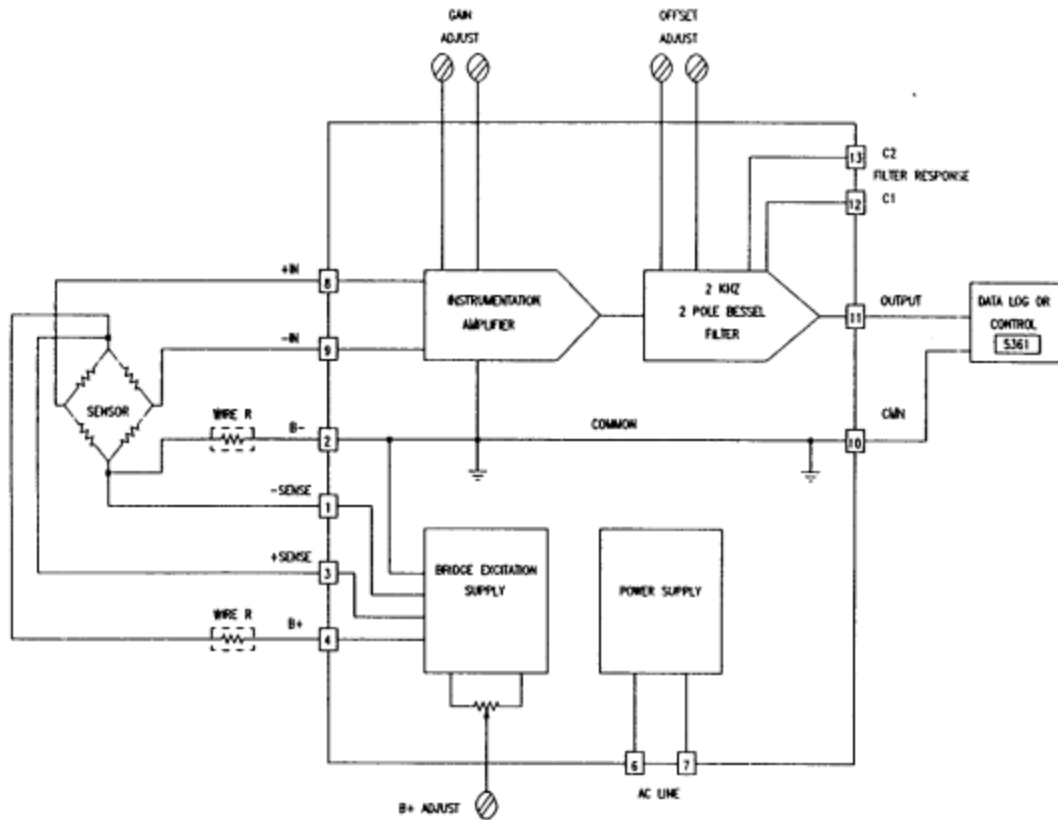
With zero loads on the strain gauge, adjust the ZERO potentiometers until the reading approaches 0.

It is required to open the enclosure to adjust both the ZERO potentiometers and the gain. The wire connections are located inside the enclosure as well.

4.1.3.3 Set-up Procedure for Omega DMD-465 WB

1. Connections

Connect the wires as indicated in Figure 6.



Terminal Strip Assignments			
Screw Terminal	Function	Screw Terminal	Function
1	-SENSE	8	+INPUT
2	B-	9	-INPUT
3	+SENSE	10	AMPLIFIER CMN
4	B+	11	AMPLIFIER OUTPUT
5	NOT USED	12	FILTER - C1
6	AC	13	FILTER - C2
7	AC		

Figure 6 Connections for Omega DMD 465-WB

2. Set excitation voltage

Connect a multi-meter to terminal 2 and 4; adjust potentiometer B+ until the reading approaches the desired excitation voltage.

3. Calibrate for zero voltage

Jumper pin 8 and 9 together, connect the inputs of DAQ box to pin 11 and 10. Adjust the COARSE OFFSET and the FINE OFFSET potentiometers until the voltage approaches 0.

4. Adjust gain

Connect the shunt resistor simulating strain desired for full scale. Adjust the COARSE GAIN and FINE gain potentiometers for the desired full scale output.

All wire connections and potentiometers of this equipment are located on the surface of the enclosure. No discrete gain setting is available, or any gain indications.

4.1.3.4 Comparison of Set-Up Procedures

Due to its lack of discrete gain setting mechanism and gain indication, and long set-up procedure, Omega is less suitable for the target laboratories. Honeywell and Tacuna are relatively easy to hook up and calibrate. For both systems, it is recommended that the connections to be extended to avoid wear of the connection pieces and reduce issues caused by bad connections in lab.

Comparing Honeywell UV-10 and Tacuna, Tacuna is more preferable. The students won't be required to open the enclosure of Tacuna if the gain setting is preset prior to the laboratory, while the students need to open the enclosure of Honeywell UV-10 to adjust the ZERO potentiometers, which might induce more complexity to managing the laboratories. Tacuna also includes internal Wheatstone bridge completion circuit, in addition to the external circuit set-up option tested in this project.

4.1.4 Selection of Alternative Signal Conditioner

Tacuna Systems Strain Gauge or Load Cell Amplifier/Conditioner Interface Manual (Tacuna) provides reliable and quiet reading, and easy set-up experience. It is most viable economical alternative to currently used signal conditioner.

Honeywell UV-10 In-line Amplifier provides satisfying performances as well, and a set-up experience slightly more complex than Tacuna. It is more costly compared to Tacuna, but also significantly more economical compared to the current system.

Out of the four signal conditioners considered, Tacuna is recommended as the top choice, while Honeywell UV-10 is recommended as the second choice.

4.1.5 Test Run of Selected Laboratories with Selected Signal Conditioners

Test runs of both laboratories are performed with both Tacuna and Honeywell UV-10, no problematic issues were encountered with the alternative equipment, and the experiments were all successful.

Sample laboratory reports of both laboratories performed with Tacuna are included in Appendix 1 and Appendix 2.

4.2 Updated Software and Instruction Materials

Updated software and instruction materials are produced for both laboratories. This section describes the improvements on the software features and the contents of the instruction material.

4.2.1 Updated Software

Updated VI for both labs added elements to the file output function. Using *Write to Measurement File* instead of *Write to Spreadsheet* adds a column for elapsed time, which makes the data more complete. The data is saved under the same directory as the program, with standardized file names and “.csv” extension.

Updated software for Strain and Pressure Measurement Laboratory incorporated some improvements of the existing VI, which include:

The LabVIEW *Formula* function is used to calculate strain, pressure and stresses, instead of *Formula Node*. The *Formula* function can process dynamic data input, while the *Formula*

Node requires dynamic data to be converted to numerical data. This enables the program to process and record all acquired data points.

The updated VI allows the user to input excitation voltage, gain, and gage factor; thus the program can be used with different hardware conditions without major alternation.

Updated software for Vibration Measurement Laboratory enables saving time domain data without alternating the program.

Step-by-step tutorials for constructing the programs are included in the updated laboratory instructions, which are contained in Appendix 3 and Appendix 4.

4.2.2 Updated Instruction Materials for Strain and Pressure Measurement Laboratory

The background part of updated instruction material for Strain and Pressure Measurement Laboratory contains brief overviews of following topics:

- Piezoresistive pressure sensor
- Stress and strain in a thin-wall cylinder
- Strain gages
 - Operating principle and application of strain gages
 - Materials and selection of strain gauges
- Wheatstone bridge

The instructions describe the experiment procedures in detail, propose data analysis tasks and discussion questions, and include optional activities and relevant reference information. Refer to Appendix 3 for the instructions document.

4.2.3 Updated Instruction Materials for Vibration Measurement Laboratory

The background part of updated instruction material for Vibration Measurement Laboratory contains brief overviews of following topics:

- Static analysis of simple cantilever beam
 - Stress, strain, and deflection associate with bending
 - Calculation of static characteristics
- Dynamic characteristics of a cantilever beam under free vibration
 - Natural frequencies of a cantilever beam under free vibration

- Mode shapes of a cantilever beam under free vibration
 - Damping factor of a cantilever beam under free vibration
- Measurement methods of dynamic characteristics
 - Fourier transformation
 - Logarithmic decrement method to determine damping frequency
 - Determining acceleration, velocity and amplitude
 - Determining elastic modulus
- Strain gages
 - Operating principle and application of strain gages
 - Materials and selection of strain gauges
- Wheatstone bridge

The instructions describe the experiment procedures in detail, propose data analysis tasks and discussion questions, and include optional activities and relevant reference information. Refer to Appendix 4 for the instructions document.

5. Conclusions and Future Work

This project examined two existing laboratories in WPI's Engineering Experiment course and made a series of recommendations and actions, in the hope of improving the educational value, student experience and the cost-effectiveness of the laboratories.

The project recommended two alternative signal conditioners to use in the laboratories. Both signal conditioners provide satisfactory performance and student experience, in addition to being economical. Adopting either of the recommended models will help reduce expenses, and enable dual or multiple axes measurements.

Software and laboratory instruction materials are updated. The software updates improved user experience of the existing program. The instruction materials contain comprehensive information to assist the students and instructors to prepare for and conduct the experiments.

A review of similar laboratories developed by other institutes is included in the background section of this report. Future effort of improving and expanding the laboratories can be focused on introducing measurement of strain multiple axes for both labs, it will provide more depth to the existing laboratories.

If the recommendations and updated materials are adopted, a brief survey can be conducted to study the effects of the changes. Inputs from instructors and students are valuable as references for future efforts to improve the laboratories.

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Appendix 1: Sample Laboratory Report for Strain and Pressure Measurement Laboratory

Abstract

In this experiment, characterization of internal pressure in a thin-walled tank (a soda can) is achieved by measurements of mechanical strains. Uncertainty analysis of characterized internal pressure is conducted with respect to parameters involved. The percentage contribution of all uncertainties to the overall uncertainty in pressure characterizations are identified in order of importance.

Description

Purpose of the Experiment

The purposes of this experiment include:

- Perform characterization of internal pressure in a thin-walled tank by measurements of mechanical strains;
- Perform uncertainty analysis of characterized internal pressures with respect to parameters involved;
- Identify, in order of importance, percentage contribution of all uncertainties to the overall uncertainty in pressure characterizations.

Experimental Procedures

In order to understand the errors in this experiment, the procedure is repeated on three soda cans of the same product.

Preparation

Find the Poisson's ratio and elastic modulus of the material used for the soda cans.

LabVIEW Program

The constructed LabVIEW program obtains amplified voltage output and transfers the output to strain, with the unit of micro-strain. A formula block calculates pressure and stress from measured strains. Measurement results are recorded. Assume 0.005 inch as can thickness.

Hardware Setup

A strain gauge is installed onto each of the three soda cans used in this experiment. Measure the diameter of the soda can.

The strain gauge is installed on the soda can surface along the circumferential direction, the height of the strain gauge location should be half of the can height.

The strain gauge is connected to the quarter bridge completion circuit. A Tacuna Systems Strain Gauge or Load Cell Amplifier/Conditioner Interface Manual (Tacuna) is used to supply excitation voltage and amplify the signal. Gain is set at 220, while the excitation is set at 5 VDC. The settings are verified by checking the switches on the circuit board. The wire connections are indicated in Figure 1.

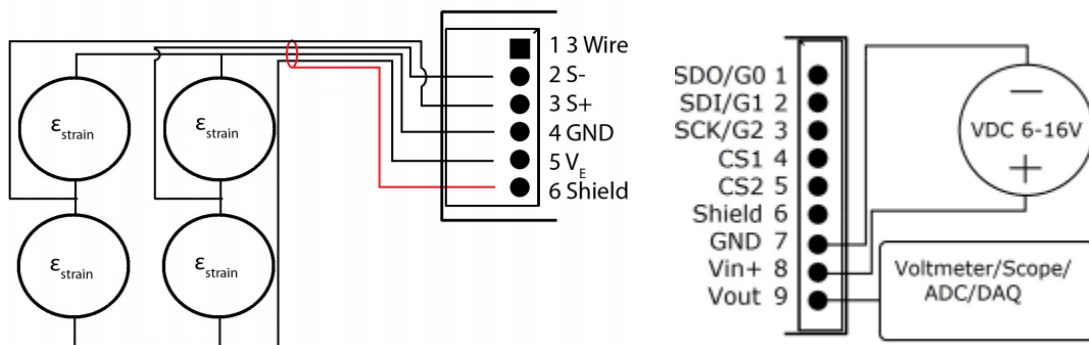


Figure 7 Connections for Tacuna Systems Strain Gauge or Load Cell Amplifier/Conditioner Interface Manual

Bridge Balancing and Verification

Make sure then NI DAQ box is turned on before starting the VI. Adjust the measurement time to 0.01 second on the LabVIEW program to view instant strain readings. Use the offset potentiometer to adjust the output voltage to 2.5V, which is half of the output range.

Connect shunt resistors parallel to the active gauge, and observe appropriate change in the strain reading.

Gently press on the tap of the can, and observe the strain readings change accordingly.

Take Measurements

Allow the amplifier to warm up for 5 minutes before taking measurements.

For each can, record strain readings for approximately 5 seconds before opening the can, and stop the VI 10 about seconds after opening the can. Use 0.005 inch for thickness during recording. After clearing out the beverage in the can and cleaning the can, measure the thickness of the can wall with a caliper. Take the average value of three measurements. Adjust the data according to the new thickness measurement.

Key Equations

The expression for internal pressure of a thin-wall vessel can be given by:

$$P = \frac{Et\epsilon_{hoop}}{r(1 - \nu/2)}$$

where E is Elastic Modulus of the material, ν is the Poisson's ratio of the material, t is the wall thickness of the vessel, r is the radius of the vessel, ϵ_{hoop} is the strain in hoop (circumferential) direction.

The stress in circumferential direction and axial direction is given by:

$$\sigma_{hoop} = \frac{Pr}{t}$$

$$\sigma_{axial} = \frac{Pr}{2t}$$

where P is internal pressure of the vessel, r is the radius, and t is the wall thickness.

Equipment List

- National Instruments USB-6229 DAQ
- Tacuna Systems Strain Gauge or Load Cell Amplifier/Conditioner Interface Manual
- Wheatstone Bridge Quarter Bridge Completion Circuit
- Vishay Strain Gage
- Shunt Resistors
- Three Cans of Soda of The Same Type

Results and Conclusions

The results of the characterization are listed in Table 1. The parameters are calculated by taking the differences of the average values prior to opening of the cans and the average values after opening the cans.

	Pressure (psi)	Hoop Strain (micro-strain)	Hoop Stress (psi)	Axial Stress (psi)
Can 1 (Fanta 355ml)	35.0	750	9.1×10^3	4.55×10^3
Can 2 (Fanta 355ml)	43.7	937	11.4×10^3	5.68×10^3
Can 3 (Fanta 355ml)	48.2	1034	12.5×10^3	6.27×10^3

Table 1 Characterization Result

From the data acquired with three Fanta 355ml cans, it can be concluded that the pressure in measured cans is in the range of 30 to 50 psi at room temperature.

Uncertainty Analysis

The parameters in Table 2 are used to perform uncertainty analysis for characterized internal pressure. The detailed process of the analysis is included in the appendix.

Parameter	Value	Uncertainty
Young's Modulus (psi)	10×10^6	5×10^4
Poisson's Ratio	0.35	0.005
Thickness (in)	0.005	5×10^{-4}
Radius (in)	1.300	2.5×10^{-3}
Strain (micro strain)	0~4000	3%

Table 2 Initial Parameters for Uncertainty Analysis

The uncertainty for measured pressure is constant at 10.4% when strain is within the specified range. The percentage contributions of each variable (when strain = 900 micro strain) are listed in the table below.

Parameter	Contribution to Uncertainty
Thickness	91.63%
Strain	8.25%
Poisson's Ratio	0.084%
Radius	0.034%
Young's Modulus	0.009%

Table 3 Percentage contribution to uncertainty

Supplemental Materials

LabVIEW Programming

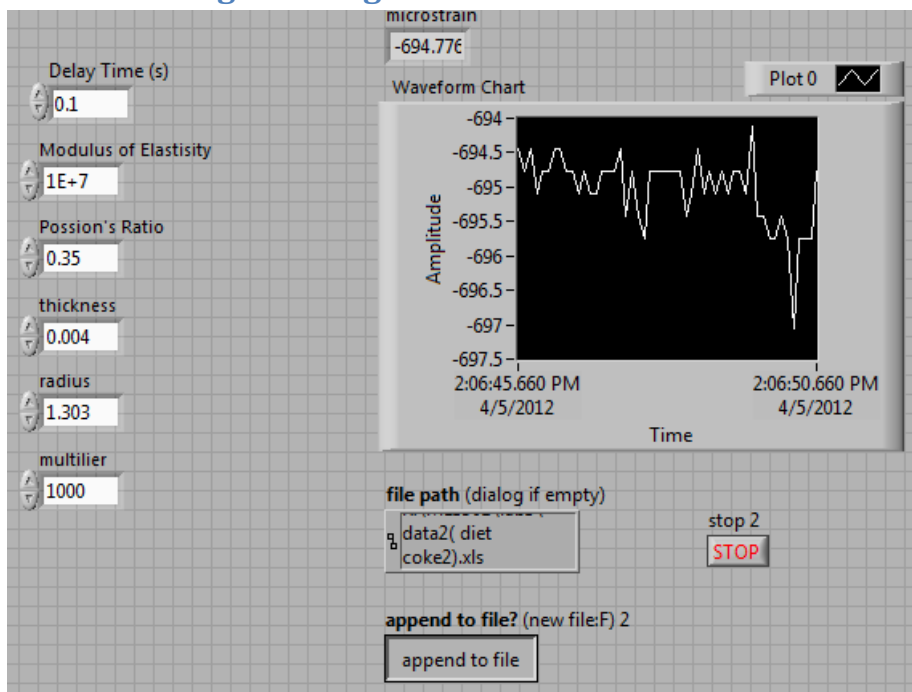


Figure 8 Front Panel of LabVIEW Program

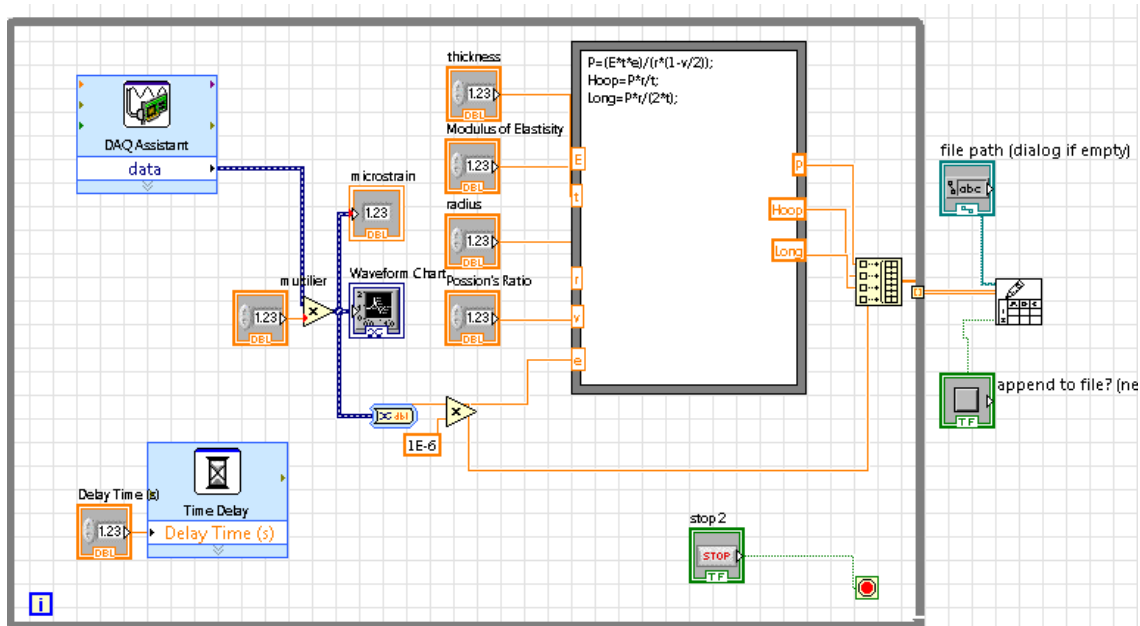


Figure 9 Block Diagram of LabVIEW Program

Uncertainty Analysis

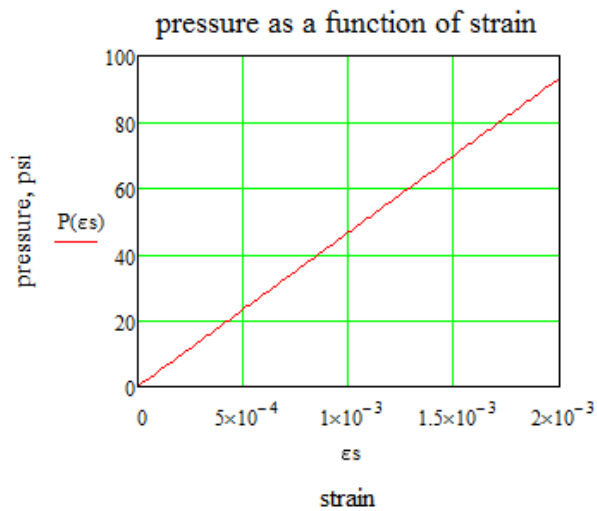
WPI ME3901 Strain and Pressure Measurement Laboratory Uncertainty Analysis

Initial Parameters:

Thickness	$t := 0.005$	inch	$\delta t := 5 \cdot 10^{-4}$
Radius	$r := 1.3$	inch	$\delta r := 5 \cdot 10^{-3}$
Elastic Modulus	$E := 10 \cdot 10^6$	$\frac{\text{lb}}{\text{in}^2}$	$\delta E := 10^4$
Poisson's Ratio	$\nu := 0.35$		$\delta \nu := 0.005$
Strain:	$\epsilon_s := 10 \cdot 10^{-6}, 20 \cdot 10^{-6} \dots 2000 \cdot 10^{-6}$		$\delta \epsilon_s(\epsilon_s) := 0.03 \cdot \epsilon_s$

Expression for Pressure:

$$P(\epsilon_s) := E \cdot t \cdot \frac{\epsilon_s}{\left(1 - \frac{\nu}{2}\right) \cdot r}$$



Uncertainty for Pressure:

Define partial derivatives functions:

$$fE(\varepsilon s) := t \cdot \frac{\varepsilon s}{\left(1 - \frac{\nu}{2}\right) \cdot r}$$

$$fEs(\varepsilon s) := E \cdot \frac{t}{\left(1 - \frac{\nu}{2}\right) \cdot r}$$

$$fthi(\varepsilon s) := E \cdot \frac{\varepsilon s}{\left(1 - \frac{\nu}{2}\right) \cdot r}$$

$$fra(\varepsilon s) := -E \cdot t \cdot \frac{\varepsilon s}{\left(1 - \frac{\nu}{2}\right) \cdot r^2}$$

$$f\nu(\varepsilon s) := \frac{1}{2} E \cdot t \cdot \frac{\varepsilon s}{\left(1 - \frac{\nu}{2}\right)^2 \cdot r}$$

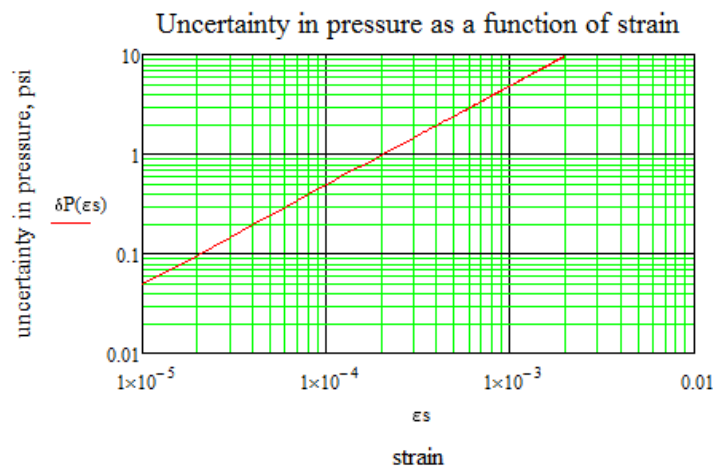
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Uncertainty in pressure as a function of strain:

$$\delta P_0(\varepsilon s) := (fE(\varepsilon s) \cdot \delta E)^2 + (fthi(\varepsilon s) \cdot \delta t)^2 + (fEs(\varepsilon s) \cdot \delta \varepsilon s)^2 + (fra(\varepsilon s) \cdot \delta r)^2 + (f\nu(\varepsilon s) \cdot \delta \nu)^2$$

$$\delta P(\varepsilon s) := \delta P_0(\varepsilon s)^{0.5}$$

$$pc(\varepsilon s) := 100 \cdot \frac{\delta P(\varepsilon s)}{P(\varepsilon s)}$$

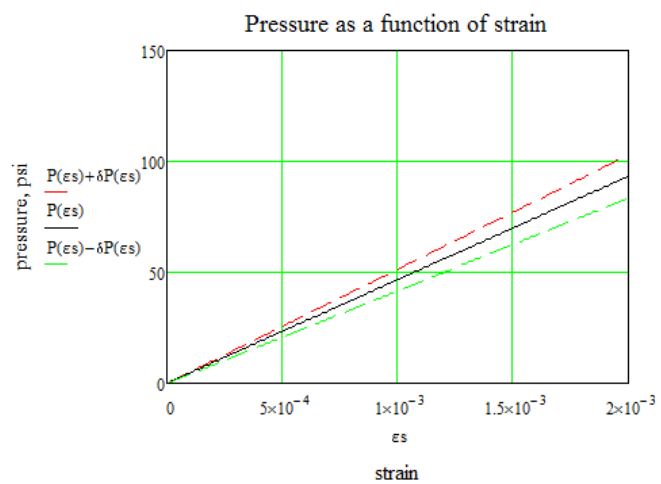
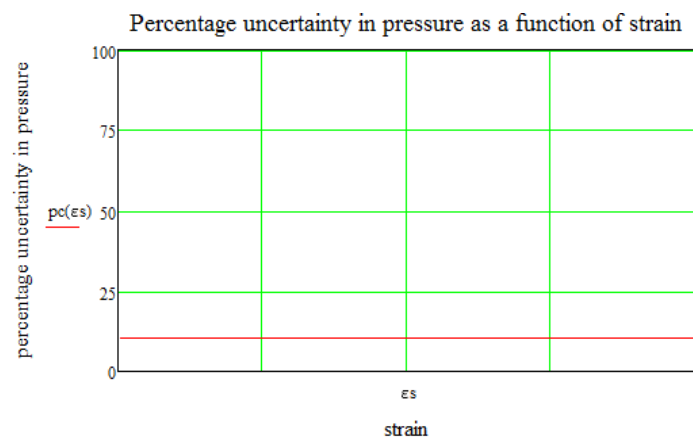


Example: evaluated when strain is 900 micro strain

$$P(900 \cdot 10^{-6}) = 41.958$$

$$\delta P(900 \cdot 10^{-6}) = 4.383$$

$$pc(900 \cdot 10^{-6}) = 10.447$$



Percentage contributions:

$$PIE(\varepsilon s) := \left(fE(\varepsilon s) \cdot \frac{\delta E}{\delta P(\varepsilon s)} \right)^2$$

$$PIthi(\varepsilon s) := \left(fthi(\varepsilon s) \cdot \frac{\delta t}{\delta P(\varepsilon s)} \right)^2$$

$$PIv(\varepsilon s) := \left(fiv(\varepsilon s) \cdot \frac{\delta v}{\delta P(\varepsilon s)} \right)^2$$

$$PIEs(\varepsilon s) := \left(fEs(\varepsilon s) \cdot \frac{\delta Es(\varepsilon s)}{\delta P(\varepsilon s)} \right)^2$$

$$PIra(\varepsilon s) := \left(fra(\varepsilon s) \cdot \frac{\delta r}{\delta P(\varepsilon s)} \right)^2$$

$$TU(\varepsilon s) := PIE(\varepsilon s) + PIthi(\varepsilon s) + PIEs(\varepsilon s) + PIra(\varepsilon s) + PIv(\varepsilon s)$$

$$PcPIE(\varepsilon s) := 100 \cdot \frac{PIE(\varepsilon s)}{TU(\varepsilon s)}$$

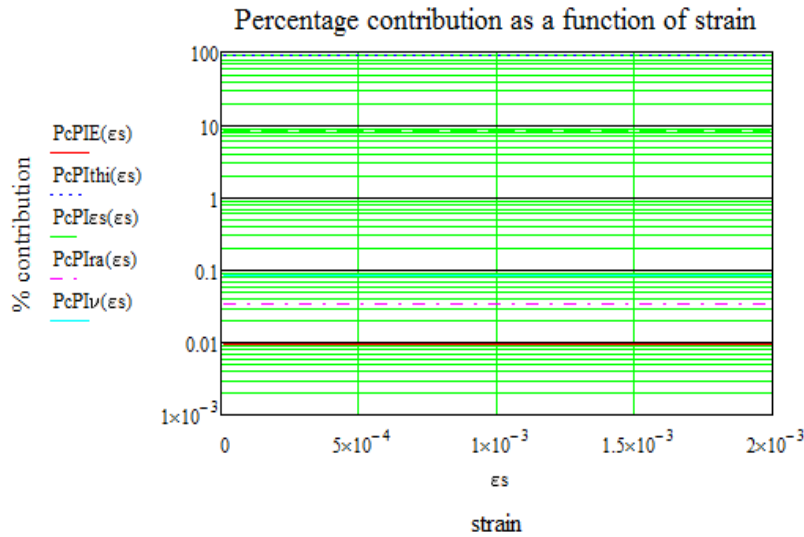
$$PcPIthi(\varepsilon s) := 100 \cdot \frac{PIthi(\varepsilon s)}{TU(\varepsilon s)}$$

$$PcPIv(\varepsilon s) := 100 \cdot \frac{PIv(\varepsilon s)}{TU(\varepsilon s)}$$

$$PcPIEs(\varepsilon s) := 100 \cdot \frac{PIEs(\varepsilon s)}{TU(\varepsilon s)}$$

$$PcPIra(\varepsilon s) := 100 \cdot \frac{PIra(\varepsilon s)}{TU(\varepsilon s)}$$

$$Check(\varepsilon s) := PcPIE(\varepsilon s) + PcPIthi(\varepsilon s) + PcPIEs(\varepsilon s) + PcPIra(\varepsilon s) + PcPIv(\varepsilon s)$$



Order of importance for the contributing factors: (evaluated when strain is 900 micro strain)

$$Check(900 \cdot 10^{-6}) = 100$$

$$PcPIthi(900 \cdot 10^{-6}) = 91.626$$

$$PcPIEs(900 \cdot 10^{-6}) = 8.246$$

$$PcPIra(900 \cdot 10^{-6}) = 0.034$$

$$PcPIv(900 \cdot 10^{-6}) = 0.084$$

$$PcPIE(900 \cdot 10^{-6}) = 9.163 \times 10^{-3}$$

Appendix 2: Sample Laboratory Report for Strain and Pressure Measurement Laboratory

Abstract

In this experiment, strain gauges are used to measure the dynamic characteristic and the material properties of cantilevers. The vibration data is analyzed to determine the parameters; the values derived from measurements are then compared with theoretical values and/or computational models.

Description

Purpose of the Experiment

The purpose of the vibration measurement experiment is to use strain gauges to measure the dynamic characteristic and the elastic material properties of cantilevers. Vibration data will be analyzed to:

- Determine the vibration amplitude, velocity, and acceleration in various units of measure;
- Determine natural frequencies;
- Measure and express damping characteristics as logarithmic decrement and percentage of critical damping;
- Compare measurements with analytical and/or computational models of a cantilever; and
- Determine elastic modulus of a cantilever

Experimental Procedures

In order to understand the errors in this experiment, the procedure is repeated on two similar cantilever beams.

Preparation

Find the elastic modulus and density of the material used for the cantilever beams.

LabVIEW Program

The constructed LabVIEW program obtains amplified voltage output and transfers the output to strain, with the unit of micro-strain. A spectral analyzer performs Fourier Transformation on measured strains. Measurement results in both time domain and frequency domains are then recorded. A sampling rate of 5k S/s is selected, and 5 seconds of data is recorded in each reading.

Hardware Setup

A strain gauge is installed onto each of the three soda cans used in this experiment. Measure the diameter of the soda can.

The strain gauge is installed on the soda can surface along the circumferential direction, the height of the strain gauge location should be half of the can height.

The strain gauge is connected to the quarter bridge completion circuit. A Tacuna Systems Strain Gauge or Load Cell Amplifier/Conditioner Interface Manual (Tacuna) is used to supply excitation voltage and amplify the signal. Gain is set at 220, while the excitation is set at 5 VDC. The settings are verified by checking the switches on the circuit board. The wire connections are indicated in Figure 1.

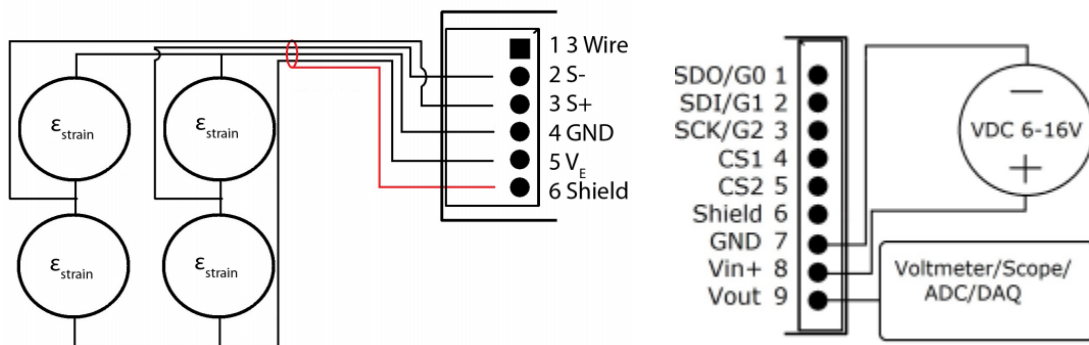


Figure 10 Connections for Tacuna Systems Strain Gauge or Load Cell Amplifier/Conditioner Interface Manual

Bridge Balancing and Verification

Make sure then NI DAQ box is turned on before starting the VI. Adjust the measurement time to 0.01 second on the LabVIEW program to view instant strain readings. Use the offset potentiometer to adjust the output voltage to 2.5V, which is half of the output range.

Connect shunt resistors parallel to the active gauge, and observe appropriate change in the strain reading.

Gently press on the tap of the can, and observe the strain readings change accordingly.

Take Measurements

Allow the amplifier to warm up for 5 minutes before taking measurements.

For each beam, continuously pluck the end of the cantilever for 30 seconds and record the vibration decay curves.

Key Equations

Strain in a Cantilever under Known Load

The strain in a cantilever beam under a known load applied at the free end is given by:

$$\varepsilon = \frac{P(L-x)t}{2EI} = \frac{6P(L-x)}{EbT^2}$$

where ε is the strain, P is the applied load, L is the length of the beam, x is the distance between the clamped end and the interested location of strain, E is the elastic modulus, b is the width of the beam, and T is the thickness of the beam.

In this experiment, the applied load is the weight of a known mass. Therefore, we have

$$P = Mg$$

where M is the mass and g is gravity.

Fundamental Frequency

The equation below describes how to predict natural frequencies of cantilever beams.

$$\omega_n = \frac{\alpha_n^2}{2\sqrt{3}} \sqrt{\frac{ET^2}{\rho L^4}} \text{ rad/s}$$

where E is elastic modulus, L is effective length of beam, ρ is the density, T is the thickness of the beam. The dimensionless wave number $\beta = 2\pi/\text{wavelength}$. β_n values for cantilever beams are: $\beta_1 L = 1.8751 = \alpha_1$, $\beta_2 L = 4.6941 = \alpha_2$, $\beta_3 L = 7.8548 = \alpha_3$, $\beta_4 L = 10.9955 = \alpha_4$, $\beta_5 L = 14.1372 = \alpha_5$, $\beta_6 L = 17.279 = \alpha_6$.

In this experiment, the effective length of the beam is close to the distance between the outer edge of the clamp and the free end of the beam.

Vibration Amplitude, Velocity, and Acceleration

The group of equations below shows the relationship how altitude and frequency determine position, velocity and acceleration during vibration.

$$y(t) = A * \sin(\omega t + \varphi)$$

$$\dot{y}(t) = \omega A * \cos(\omega t + \varphi)$$

$$\ddot{y}(t) = -\omega^2 A * \sin(\omega t + \varphi)$$

The altitude is equivalent to maximum deflection at vibration peaks. The amplitude can be given by:

$$A = w_{\max} = \frac{\varepsilon}{3T(L-x)}$$

Therefore, the maximum velocity and maximum acceleration can be expressed as:

$$v_{\max} = \omega A = \frac{\omega \varepsilon}{3T(L-x)}$$

$$a_{\max} = \omega^2 A = \frac{\omega^2 \varepsilon}{3T(L-x)}$$

Damping Ratio

A common method for analyzing the damping of an underdamped oscillation is the logarithmic decrement method, for which the following relationships apply.

$$\ln\left(\frac{y_i}{y_{i+n}}\right) = n\delta$$

$$\zeta = \frac{\delta}{\sqrt{(2\pi)^2 + \delta^2}}$$

$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}}$$

where y_i is the amplitude of peak i (i is an integer counting each peak), n is the number of cycles being considered, δ is the log decrement, ω_n is the undamped natural frequency, and ω_d is the damped natural frequency. Both frequencies are in radian per second. Note, it is assumed that object oscillates about zero. If there is an offset in y , the y_i amplitude must be defined relative to that offset.

Elastic Modulus

From the expression for natural frequency, the expression for elastic modulus can be derived as:

$$E = \frac{12\omega_n^2 \rho L^4}{\alpha_n^4 T^2}$$

where ω_n is n^{th} natural frequency, ρ is density, L is effective length, T is thickness, and α_n values for cantilever beams are $\alpha_1=1.8751$, $\alpha_2=4.6941$, $\alpha_3=7.8548$, $\alpha_4=10.99557$, $\alpha_5=14.1372$, $\alpha_6=17.279$.

Equipment List

- National Instruments USB-6229 DAQ
- Tacuna Systems Strain Gauge or Load Cell Amplifier/Conditioner Interface Manual
- Wheatstone Bridge Quarter Bridge Completion Circuit
- Vishay Strain Gage
- Shunt Resistors
- Three Cans of Soda of The Same Type
- A Cantilever Beam

Results and Conclusions

Initial Measurements and Research

The beams used in this experiment are made with 6061 Aluminum. The Modulus of Elasticity and density of the material are listed in Table 1.

Parameter	Value (SI Unit)	Value (English Unit)
Young's Modulus (E)	68.9GPa	10^4 ksi
Density (ρ)	2.7×10^3 Kg/m ³	0.0975 lb/in ³

Table 1 Material Property of 6061 Aluminum

The measured beam dimensions and the gauge location for the beam is listed in Table 2. Effective length is the distance from the outer edge of the aluminum plate (between the c-clamp and the beam) to the free end of the beam; the gauge location is defined as distance from the center of the gauge to the outer edge of the alumni plate.

	Value (SI Unit)	Value (English Unit)
Thickness (T)	3×10^{-3} m	1/8 in
Width (b)	0.0255m	1 in
Effective Length (L)	0.270m	10 5/8 in
Gauge Location (x)	0.0255m	1 in

Table 8 Dimensions of Beam 1

Verification of Correct Installation

To verify the correct installation, a weight attached close to the free end on the beam. The analytical strain at strain gauge location is derived and compared with measured values, as shown in Table 3. The error is within 3% compared to the theoretical strain derived, therefore, the installation is correct.

Theoretical Strain (micro strain)	Measured Strain (micro strain)	Error
240.5	235	2.5%

Table 3 Comparison of Theoretical Strain and Measured Strain

Determine Natural Frequencies

The fundamental frequency is determined by finding the frequency of the first “peak” in the power spectrum plot. Figure 1 shows results of Fourier Transformation from strain data obtained from a vibrating beam.

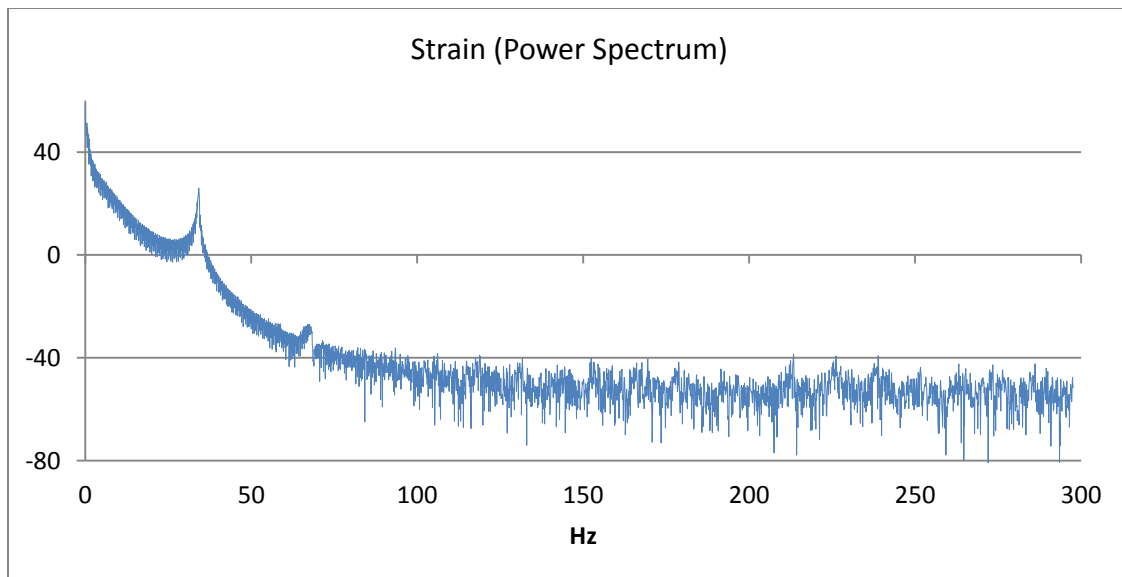


Figure 11 Power Spectrum of Strain

The fundamental frequencies of the two beams used in this experiment are determined with this method. The measured value is then compared with theoretical undamped natural frequency in the Table 4.

Theoretical Natural Frequency (Hz)	Measured Natural Frequency (Hz)
35.5	35.3

Table 4 Comparison of Measured Fundamental Frequency and Theoretical Fundamental Frequency

Determine the Vibration Amplitude, Velocity, and Acceleration

Figure 2 shows the plot for the strain measured from one of the beams during free vibration.

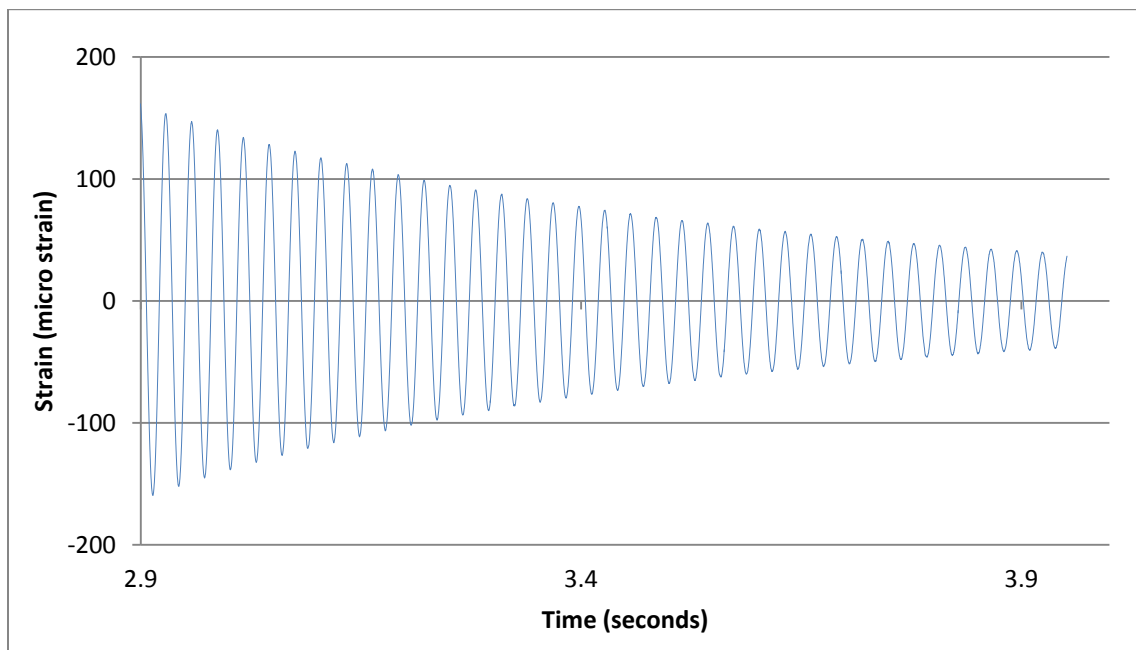


Figure 12 Strain Measured from Vibrating Beam

The vibration amplitude, or maximum displacement at each peak, is derived from the peak strain in the first vibration cycle and the beam dimensions with the relationship below.

$$A_{\text{peak}} = w_{\text{max}} = \frac{\varepsilon}{3T(L - x)}$$

The maximum velocity and acceleration is derived with the maximum displacement and acceleration, as shown in Table 6.

	Acceleration (g)	Acceleration (m/s²)	Velocity (m/s)	Amplitude (m)
Peak	6.6	64.7	1.84	0.052
Peak-To-Peak	13.2	129.4	3.67	0.104
RMS	4.7	45.8	1.30	0.036

Table 5 Maximum Acceleration, Velocity and Displacement

Measure and Express Damping Characteristics

Using logarithmic decrement method, the damping ratio for the beam is determined from measured data, as shown in Table 7.

Logarithmic Decrement	Damping Ratio
0.042	0.0067

Table 6 Damping Ratio

Predict Elastic Modulus.

The derived elastic modulus from the measurements and analytical prediction is listed below. Predicted elastic modulus of the cantilever beam is 68GPa, the errors are close to theoretical value.

	Theoretical Elastic Modulus (GPa)	Measured Elastic Modulus (GPa)	Error
Beam 1	68.9	68.2	1%

Table 7 Comparison of Theoretical Elastic Modulus and Measured Elastic Modulus

Uncertainty Analysis

The uncertainty of natural frequency is 0.05 Hz, which is limited by the resolution of spectral analyzer in LabVIEW program.

Uncertainty analyses with respect to the parameters are performed for damping ratio and amplitude. Parameters used in the uncertainty analyses are shown in Table 9. Thickness, length and gauge location are measured with a ruler, which has a least significant digit of $\frac{1}{16}$ inch. Therefore, the uncertainty for these three items is $\frac{1}{32}$ inch, or 7.94×10^{-4} meter. An uncertainty of 3% is assumed for measured strains.

	Value	Uncertainty
Thickness (m)	3.175×10^{-3}	7.94×10^{-4}
Length (m)	0.27	7.94×10^{-4}
Gauge Location (m)	0.0254	7.94×10^{-4}
Natural frequency (Hz)	35	0.05
Strain (micro strain)	0 to 10^4	3%

Table 8 Initial Values for Uncertainty Analysis

When logarithmic decrement is 0.043, the uncertainty for damping ratio is 9.9%. Uncertainty in strain measurement is the only contributing factor.

The uncertainty for amplitude 25.2% when strain is within the range defined in Table 9. The contributions of each factor are listed in Table 10.

	Contribution
Thickness	98.5%
Strain	1.4%
Length	0.02%
Gauge location	0.02%

Table 9 Contributions of Parameters to Uncertainty in Amplitude

The steps of the uncertainty analysis are included in the Appendix.

Supplemental Materials

LabVIEW Program

Figure 3 shows the front panel of the LabVIEW program used in this experiment. Figure 4 shows the block diagram of the LabVIEW program.

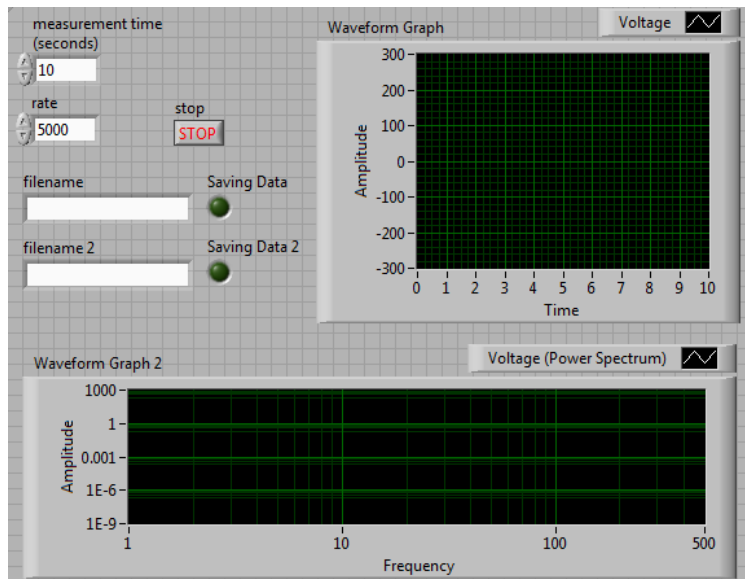


Figure 13 Front Panel of LabVIEW Program

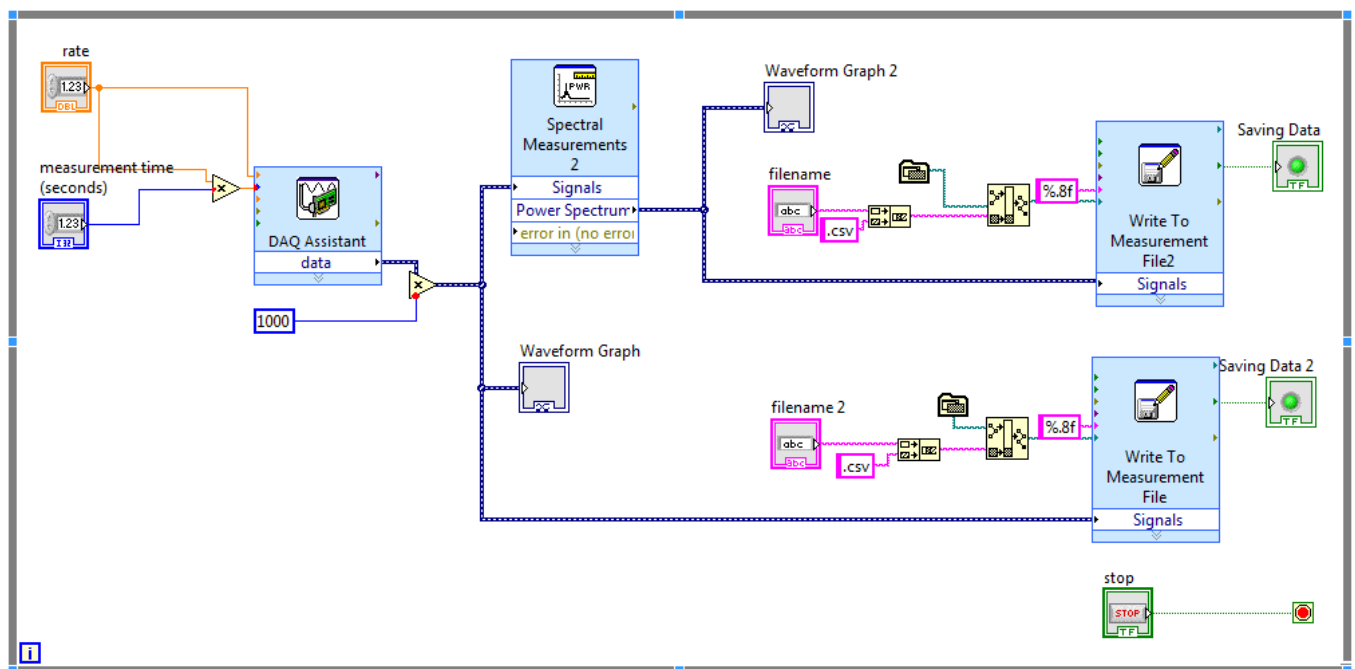


Figure 14 Block Diagram of LabVIEW Program

Uncertainty Analysis

WPI ME3901 Vibration Measurement Laboratory Uncertainty Analysis

Initial Parameters:

Thickness:	$T := \frac{1-0.0254}{8}$	m	$\delta T := \frac{1-0.0254}{32}$
Length:	$L := 0.27$	m	$\delta L := \frac{0.0254}{32}$
Gauge location	$x := 0.0254$	m	$\delta x := \frac{0.0254}{32}$
Strain:	$\epsilon s := 0 \cdot 10^{-6}, 10 \cdot 10^{-6} \dots 400 \cdot 10^{-6}$		$\delta \epsilon s(\epsilon s) := 0.03 \epsilon s$
Natural frequency	$w := 35$	Hz	$\delta w := 0.05$

Expressions for Dynamic Characteristics:

Logarithmic Decrement

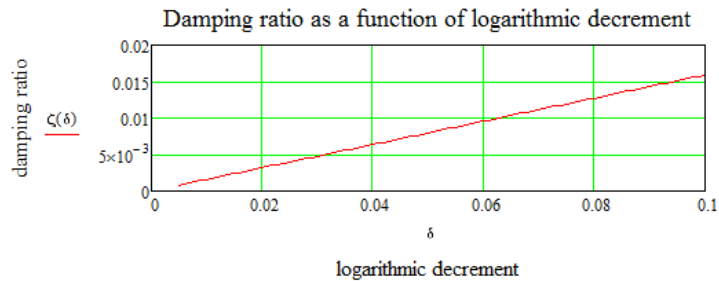
$$n := 10$$

$$\delta = 1/n \cdot \ln(\epsilon_i/\epsilon_{i+n})$$

$$\delta := 0.005, 0.006 \dots 0.1$$

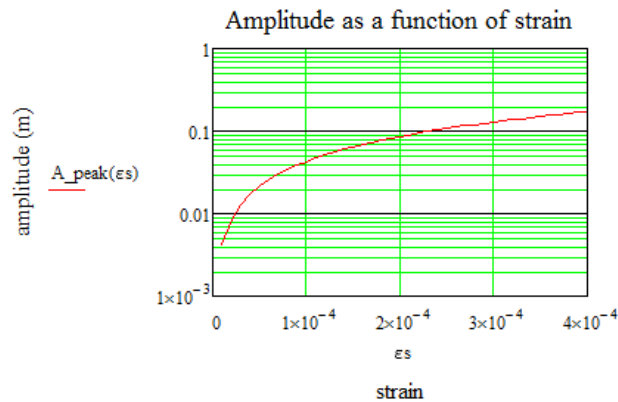
Damping Ratio

$$\zeta(\delta) := \frac{\delta}{(4 \cdot \pi^2 + \delta^2)^{0.5}}$$



Amplitude

$$A\_peak(\epsilon s) := \frac{\epsilon s}{3T \cdot (L - x)}$$



Uncertainty for Logarithmic Decrement:

$$d\delta(\varepsilon s) := \left(\frac{\delta \varepsilon s(\varepsilon s)}{n \cdot \varepsilon s} \right)^2 + \left(\frac{-\delta \varepsilon s(\varepsilon s)}{n \cdot \varepsilon s} \right)^2$$

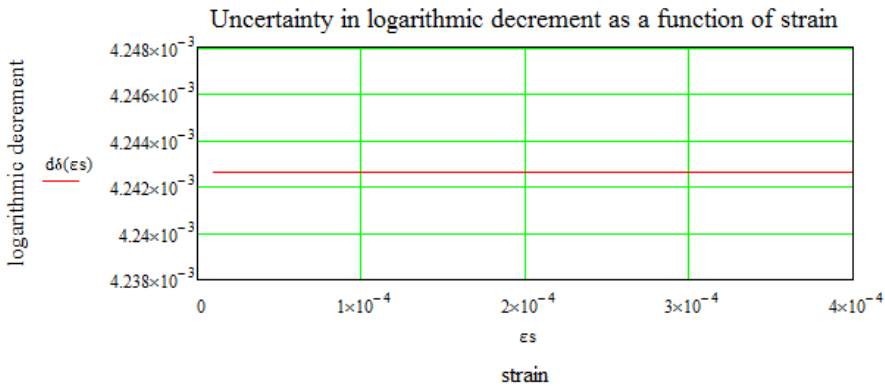
$$d\delta(\varepsilon s) := d\delta l(\varepsilon s)^{0.5}$$

(uncertainty in logarithmic decrement as a function as strain)

Example: evaluated at 150 micro strain

$$d\delta(150 \cdot 10^{-6}) = 4.243 \times 10^{-3}$$

Uncertainty for logarithmic decrement is 0.004243 for any strain when n=10. Uncertainty for strain is the only contributing factor.



Uncertainty for Damping Ratio:

$$d\zeta(\delta) := d\delta(1000 \cdot 10^{-6}) \cdot 4 \cdot \frac{\pi^2}{(\delta^2 + 4\pi^2)^{1.5}}$$

$$pc(\delta) := \frac{100 \, d\zeta(\delta)}{\zeta(\delta)}$$

Example: evaluated when logarithmic decrement is 0.043

$$d\zeta(0.043) = 6.752 \times 10^{-4}$$

$$pc(0.043) = 9.866$$

Uncertainty for logarithmic decrement is the only contributing factor to the uncertainty for damping ratio.

Since the logarithmic decrement is complete dependent on the uncertainty for strain, the only contributing factor for damping ratio is the uncertainty for strain.

At high level of logarithmic decrement ($\delta=0.08$):

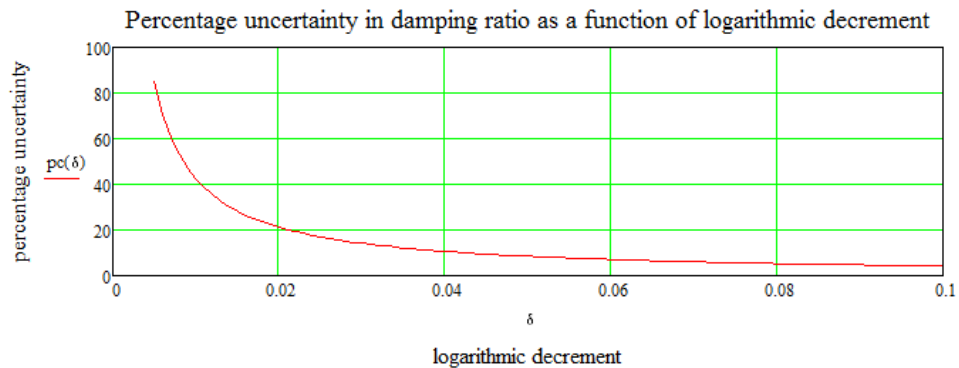
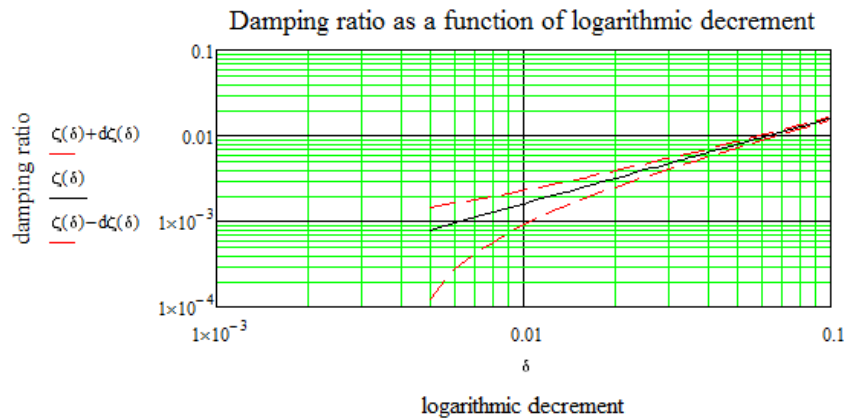
$$d\zeta(0.08) = 6.751 \times 10^{-4}$$

$$pc(0.08) = 5.302$$

At low level of logarithmic decrement ($\delta=0.01$):

$$d\zeta(0.01) = 6.752 \times 10^{-4}$$

$$pc(0.01) = 42.426$$



Uncertainty for Amplitude:

$$f_{\varepsilon s} := \frac{1}{3T \cdot (L - x)}$$

$$f_L(\varepsilon s) := \frac{\varepsilon s}{3T \cdot (L - x)^2}$$

$$f_T(\varepsilon s) := \frac{\varepsilon s}{3T^2 \cdot (x - L)}$$

$$f_x(\varepsilon s) := \frac{\varepsilon s}{3T \cdot (x - L)^2}$$

$$dA1(\varepsilon s) := (f_{\varepsilon s} \cdot \delta \varepsilon s)^2 + (f_L(\varepsilon s) \cdot \delta L)^2 + (f_T(\varepsilon s) \cdot \delta T)^2 + (f_x(\varepsilon s) \cdot \delta x)^2$$

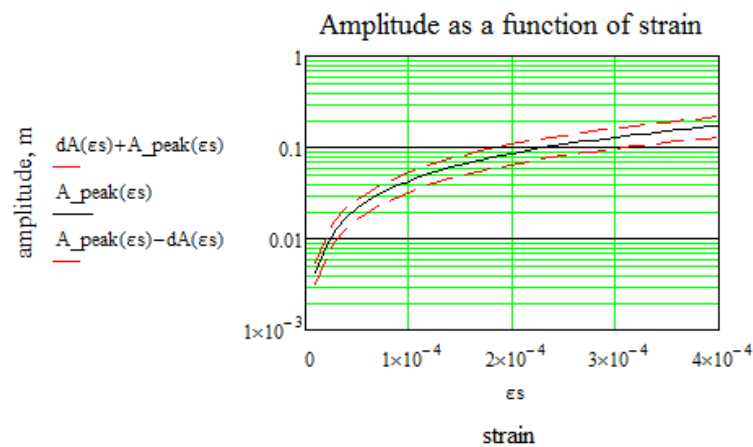
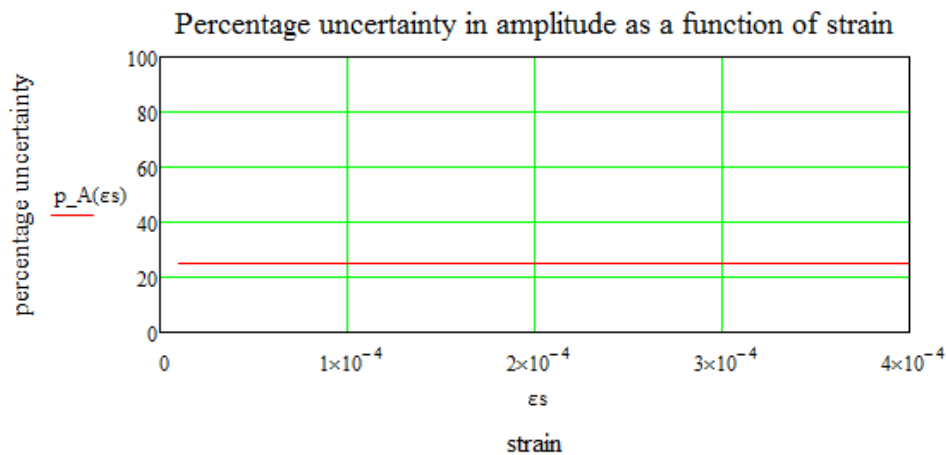
Uncertainty as a function of strain:

$$dA(\varepsilon s) := dA1(\varepsilon s)^{0.5}$$

$$p\_A(\varepsilon s) := \frac{100 \, dA(\varepsilon s)}{A\_peak(\varepsilon s)}$$

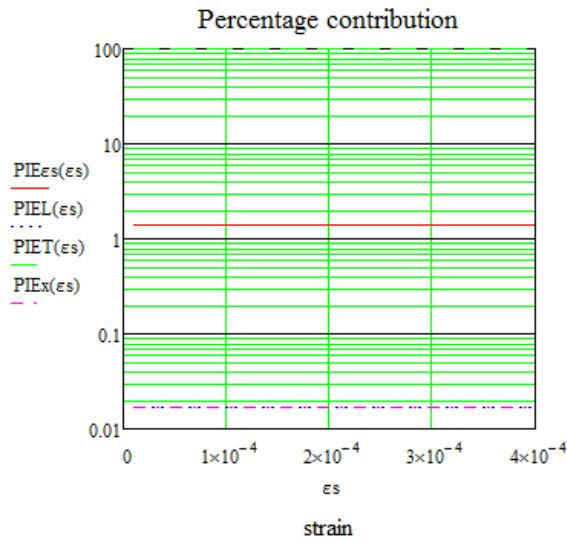
Example: strain at 100 micro strains

$$p\_A(150 \cdot 10^{-6}) = 25.184$$



Percentage Contribution:

$$\begin{aligned} \text{PIE}_{\varepsilon s}(\varepsilon s) &:= 100 \left(\text{f}_{\varepsilon s} \cdot \frac{\delta \varepsilon s(\varepsilon s)}{dA(\varepsilon s)} \right)^2 & \text{PIEL}(\varepsilon s) &:= 100 \left(\text{f}_L(\varepsilon s) \cdot \frac{\delta L}{dA(\varepsilon s)} \right)^2 \\ \text{PIET}(\varepsilon s) &:= 100 \left(\text{f}_T(\varepsilon s) \cdot \frac{\delta T}{dA(\varepsilon s)} \right)^2 & \text{PIEx}(\varepsilon s) &:= 100 \left(\text{f}_x(\varepsilon s) \cdot \frac{\delta x}{dA(\varepsilon s)} \right)^2 \end{aligned}$$



Order of importance for contributing factoes. evaluated when strain is 150 micro strains

$$\text{PIET}(150 \cdot 10^{-6}) = 98.548$$

$$\text{PIE}_{\varepsilon s}(150 \cdot 10^{-6}) = 1.419$$

$$\text{PIEL}(150 \cdot 10^{-6}) = 0.017$$

$$\text{PIEx}(150 \cdot 10^{-6}) = 0.017$$

Appendix 3: Instructions for Strain and Pressure Measurement Laboratory

Laboratory: Strain and Pressure Measurement

1. OBJECTIVES

The objectives of this laboratory include:

- ❖ Perform characterization of internal pressure in a thin-walled tank by measurements of mechanical strains;
- ❖ Perform uncertainty analysis of characterized internal pressures with respect to parameters involved;
- ❖ Identify, in order of importance, percentage contribution of all uncertainties to the overall uncertainty in pressure characterizations;

2. BACKGROUND

A thin walled cylinder has a wall thickness smaller than 1/10 of the cylinder's radius. In this case, only the membrane stresses are considered and the stresses are assumed to be constant throughout the wall thickness.

The ASME boiler codes require continuous monitoring of pressure in thin walled pressure vessels. In certain processes, use of mechanical pressure gauge or electrical pressure transducer to monitor the pressure is impractical, as the diaphragm can become encrusted with chemical products quickly. Therefore, a new method is required.

2.1 Piezoresistive Pressure Sensor

As shown in figure below, a pressure transducer consists of a diaphragm and four strain gages installed on the metal film attached to the diaphragm. Note that strain gauges R_1 and R_3 are in the radial direction and strain gauges R_2 and R_4 are in the direction transversal to the radius. Therefore, when pressure increases, the resistance of R_1 and R_3 increases and R_2 and R_4 decreases. The four strain gages form a Wheatstone bridge, as shown in figure below. The change in output voltage of a pressure transducer is directly proportional to the change in pressure. The relationship between output voltage (V_{out}) and the excitation voltage (V_{in}) is shown equation 1.

$$V_{out} = V_{in} \left(\frac{R_1}{R_1 + R_4} - \frac{R_2}{R_2 + R_3} \right) \quad \text{Eq.1}$$

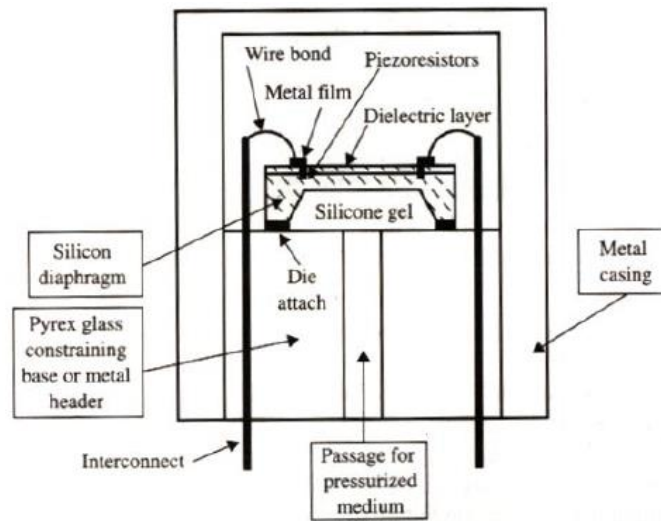


Figure 15a Cross Section of a Pressure Transducer

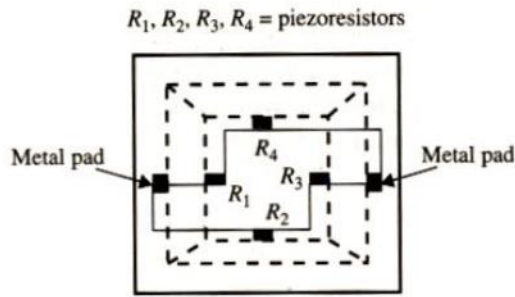


Figure 1b Top View of a Pressure Transducer

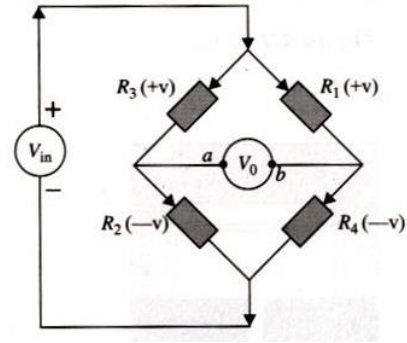


Figure 1c Circuit Diagram of a Wheat Stone Bridge

2.2 Stress and Strain in a Thin-Wall Cylinder

For vessels with a wall thickness of no more than one-tenth of its radius, the wall can be treated as a surface. The law of LaPlace holds for fluid or gas filled hollow objects with radius r . For cylinders, the internal pressure acts on them to develop a force along the axis of the cylinder.

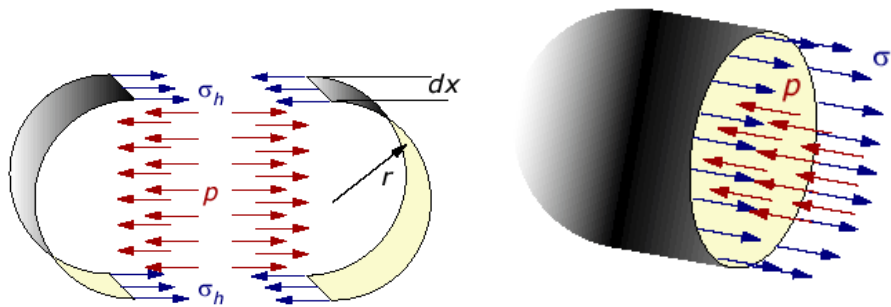


Figure 16 Static equilibrium in cross-sections of a thin-walled cylinder

To determine the hoop stress σ_{hoop} , equilibrium in the radial direction should be examined. As illustrated in figure 2, the pressure acts to “pull the two halves apart”, while the hoop stress balances the effect. The hoop stress yields,

$$2\sigma_{hoop}t \, dx = P \, 2r \, dx \quad \text{Eq.1}$$

Therefore,

$$\sigma_{hoop} = \frac{Pr}{t} \quad \text{Eq.2}$$

Similarly, in the axial direction, the pressure acts to push the two halves apart, while axial stress balances the effects, as shown in figure 3. The axial stress yields,

$$2\pi r * \sigma_{axial} * t = P * \pi * r^2 \quad \text{Eq.3}$$

$$\sigma_{axial} = \frac{Pr}{2t} \quad \text{Eq.4}$$

The Hooke's Law states that stress in the can is proportional to the strain. The relationship in this case can be expressed as:

$$\varepsilon_{Hoop} = \frac{\sigma_{hoop} - \nu * \sigma_{axial}}{E} \quad \text{Eq.5}$$

where E is Elastic Modulus of the material, and ν is Poisson's ratio of the material.

With Eq.2, Eq.4, and Eq.5, the relationship between pressure and strains can be derived as:

$$P = \frac{Et\varepsilon_{hoop}}{r(1 - \nu/2)} \quad \text{Eq.6}$$

2.3 Basics of Strain Gages

2.3.1 Operating Principle and Application of Strain Gages

Strain-gauge sensor is one of the most commonly used means of load, weight, and force detection. Strain gauges are frequently used in mechanical engineering research and development to measure the stresses generated by machinery, and in Aircraft component testing to structural measure stress of members, linkages, and any other critical component of an airframe.

A strain gauge operates on the principle that the electrical resistance of a wire changes when the length of the wire varies. It is used for measuring deformations in solid bodies. The strain experienced by the sensor is directly proportional to the change in resistance of the gauge used, as shown in Eq 7. When unstressed, usual strain gauge resistances range from 30 Ohms to 3 kOhms.

$$R = \rho \frac{L}{A} \quad \text{Eq.7}$$

An ideal strain gage is small in size and mass, low in cost, easily attached, and highly sensitive to strain but insensitive to ambient or process temperature variations. The ideal strain gauge would undergo change in resistance only because of the deformations of the surface to which the sensor is coupled. However, in real applications, there are many factors which influence detected resistance such as

temperature, material properties, the adhesive that bonds the gage to the surface, and the stability of the metal.

The strain sensitivity, which is also known as the gage factor (GF) of the sensor, is given by:

$$F = \frac{dR/R}{\varepsilon_x} \quad \text{Eq.8}$$

where R is the resistance of the gauge without deformation, dR is the change in resistance caused by strain, and ε_x is the strain to be measured. Therefore, the strain can be expressed as:

$$\varepsilon_x = \frac{1}{F} \frac{dR}{R} \quad \text{Eq.9}$$

2.3.2 Materials and Selection of Strain Gauges

Typical materials for strain gages include: constantan (copper-nickel alloy), nichrome v (nickel-chrome alloy), platinum alloys (usually tungsten), isoelastic (nickel-iron alloy), karma-type alloy wires (nickel-chrome alloy), foils, and semiconductor materials. The most popular alloys for strain gages are copper-nickel alloys and nickel-chromium alloys.

Temperature change can affect the internal structure of strain-sensing material, and also can amend properties of the material of the surface the strain gage is attached to. When there is a temperature change while a measurement is being made, the effects can cause large errors in data unless proper precautions are taken.

Each material has unique reaction to temperature change, as illustrated in figure below. Variation in expansion coefficients between the gage and base materials may cause dimensional changes in the sensor element. Therefore, it is a good practice to select strain gauge made of same type of material as the base structure.

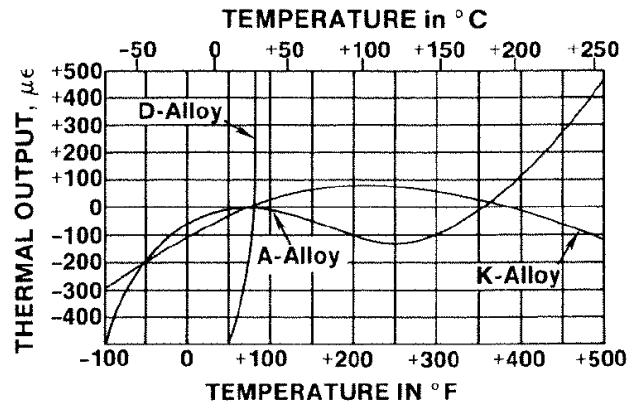


Figure 17 Temperature Effects on Thermal Output of Strain Gauges

Strain gauge's product name contains all critical information needed to select appropriate gauge. The meanings of each part of the name are shown in Figure 18 below. While Figure 19 shows key information of the type of strain gauge selected for this experiment.

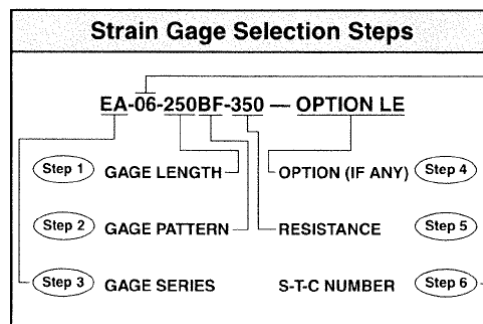


Figure 18 Strain Gage Selection Steps

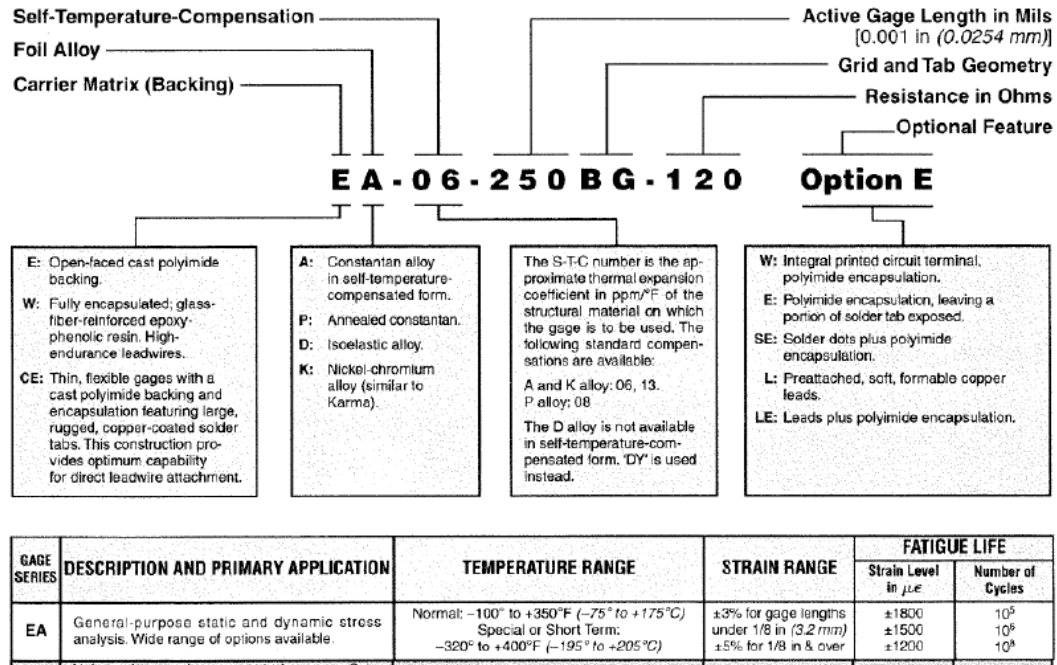


Figure 19 Crucial Information of Strain Gauge Selected

2.4 Basics of Wheatstone bridge

A Wheatstone bridge is an electrical circuit used to measure an unknown electrical resistance (from 1 Ω to 1M Ω) by balancing two legs of a bridge circuit, one leg of which includes the unknown component. A circuit diagram of Wheatstone bridge is shown in figure below, where the battery (symbol “E” serves as an excitation source, and the output is measured by a potentiometer “G”).

A “balanced” bridge is one with potential difference between B and D is equal to zero. Balance is sensed by closing switch S2 and measuring output current and voltage – to be near zero. Voltage drop across R2 is equal to voltage drop across R1, since voltage difference between B and D is equal to zero. Therefore,

$$R_x = \frac{R_1 R_3}{R_2} \quad \text{Eq.10}$$

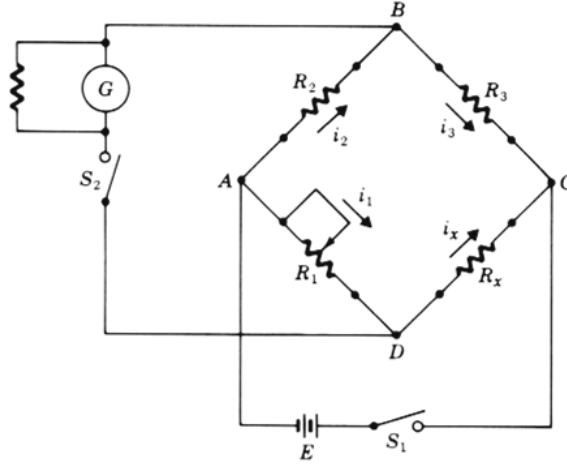


Figure 6 circuit diagram of Wheatstone bridge

When the bridge is unbalanced, equivalent resistance of the circuit is,

$$R = \frac{R_1 * R_4}{R_1 + R_4} + \frac{R_2 * R_3}{R_2 + R_3} \quad \text{Eq.10}$$

When the circuit is viewed as a circuit divider, the output voltage is,

$$E_g = \left(\frac{E}{R_1 + R_4} \right) * R_1 - \left(\frac{E}{R_2 + R_3} \right) * R_2 = E * \frac{R_1 R_3 - R_2 R_4}{(R_1 + R_4)(R_2 + R_3)} \quad \text{Eq.11}$$

When the resistance of R_4 changes by a small amount (ΔR_4), the new output voltage is,

$$E_g + \Delta E_g = E * \frac{R_1 R_3 - R_2(R_4 + \Delta R_4)}{(R_1 + R_4 + \Delta R_4)(R_2 + R_3)} = E \left(\frac{1 + \frac{\Delta R_4}{R_4} - \frac{R_3 R_1}{R_4 R_2}}{\left(1 + \frac{R_1}{R_4} + \frac{\Delta R_4}{R_4}\right) \left(1 + \frac{R_3}{R_2}\right)} \right) \quad \text{Eq.12}$$

If the bridge was originally balanced ($E_g = 0, R_1 = R_2 = R_3 = R_4$), then we have,

$$\Delta E_g = \frac{E * \frac{\Delta R_4}{R_4}}{4 + 2 \left(\frac{\Delta R_4}{R_4} \right)} \quad \text{Eq.13}$$

Since change in resistance is really small ($\Delta R_4 \ll 1$), the change in output voltage is,

$$\Delta E_g = \frac{E * \Delta R_4}{4 R_4}$$

or,

$$\Delta E_g = \frac{E * \Delta R}{4 R} \quad \text{Eq.14}$$

3. PROCEDURES

In order to estimate the internal pressure of soda cans, the procedures of this experiment include research for relevant data, hardware set-up, construction of LabVIEW program, signal conditioning, taking measurements, and data analysis. The measurements should be repeated on 3 soda cans.

The information acquired from research and part of the measurement process should also be used to produce uncertainty analysis and the contribution of each parameter to total uncertainty.

3.1 Preparation

3.1.1 Research for Relevant Parameters

Research for the parameters below, and provide references:

- Standard dimension of soda cans: diameter and thickness.
- Material property of soda cans: material type, elastic modulus, and Poisson's ratio.
- Common internal pressure range of soda cans.

Document 1 includes relevant data and some information on the materials of soda cans.

3.1.2 Understand the Effects of Gain in the Signal Conditioner

Calculate the amplifier gain required to amplify the output of the Wheatstone bridge so that you get 1 mV/micro-strain.

Recall Gage factor defined in eq.8,

$$F = \frac{dR/R}{\epsilon_x}$$

Measured strain can be expressed as

Eq.15

$$\epsilon_x = \frac{\Delta R}{FR_x}$$

Recall the expression of change in bridge output voltage caused by a small change in resistance from eq.14,

$$\Delta E_g = \frac{E \Delta R}{4R}$$

From eq.15 and eq.14, the relationship between measured strain and change in output can be found as,

Eq.16

$$\Delta E_g = \frac{F}{4} E \varepsilon_x$$

To achieve an output signal of 1mV per $\mu\varepsilon$, the gain (G) needs to satisfy:

$$\frac{G \Delta E_g}{\varepsilon_x} = \frac{1 \times 10^{-3} V}{1 \times 10^{-6}} = 1 \times 10^3 V$$

Therefore,

$$G = 1 \times 10^3 V \times \frac{4}{EF} \quad \text{Eq.17}$$

For this experiment, gage factor (F) is $2.095 \pm 0.5\%$.

3.1.3 Calculate Strain Simulated by Shunt Resistors

Calculate the strain simulated by shunt resistors.

The connected shunt resistors are parallel to the gage, the equivalent resistance is:

$$R_{eq} = \frac{R_x R_{cal}}{R_x + R_{cal}} \quad \text{Eq.18}$$

Therefore,

$$\varepsilon_{eq} = \frac{\Delta R}{F * R_x} = \frac{R_{eq} - R_x}{F * R_x} = \frac{R_x}{F(R_x + R_{cal})} \quad \text{Eq.19}$$

Gage factor is $2.095 \pm 0.5\%$ for the gauge chosen for this experiment. Resistance without deformation is 120Ω .

3.2 Setup

For measurement of strain via a bonded resistance strain gage, it must be connected to an electrical measuring circuit which can accurately measure changes in resistance. Strain-gage transducers usually employ four strain elements electrically connected to form a Wheatstone bridge. This bridge circuit compensates for temperature effects. Quarter bridge strain gauge circuits are appropriate for this purpose.

3.2.1 Hardware setup

1. Prepare 3 cans of sodas; they should be of the exact same brand and product.
2. Strain gauges should be mounted in hoop direction of the soda can.

Besides strain gauge and the cans, material needed for attaching the gauge to a surface include: sand paper, degreaser/alcohol, conditioner, neutralizer solutions, cotton balls & swabs, one-side sticky tape, adhesive, low-impedance strain gage wire (about 15 “), and soldering material. The steps of are explained below.

- 1) Degreasing: wipe the surface with degreaser or alcohol to remove oil, grease, organic taminants and soluble chemical residues.
- 2) Surface abrading: sand the surface with sand paper, in order to remove loosely bonded adherents (scalc, rust, paint, coating, oxides, etc.) and develop a surface texture suitable for bonding.
- 3) Mark layout lines: mark the planned positions to attach strain gauges.
- 4) Apply neutralizer to the surface, alcohol works as well.
- 5) Mount on tape: secure strain gauge to the surface with tape, before applying adhesive. When mounting the gauge to the tape, make sure that the side of the gage with soldering terminals should be facing the tape, or “facing up” from the surface.

Carefully remove the strain gauge from its package with tweezers, make sure the strain gauge stay chemically clean. Attach one end of a 4-to-6 inch tape to the surface, carefully attach the strain gage to the tape with tweezers, then pick the gage up by lifting the tape at a shallow angle until the tape comes free with the gage and terminal attached. See figure below for illustration of this step.

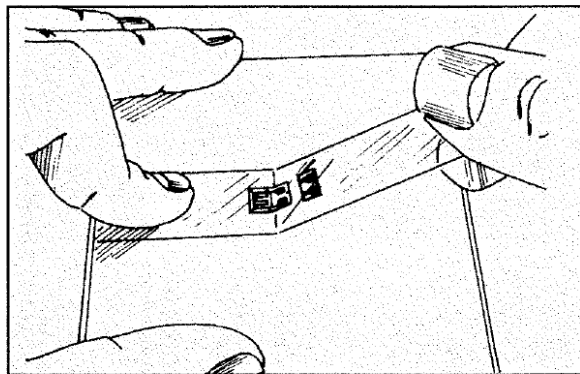


Figure 20 Mount the strain gauge on tape

- 6) Position the tape: position the gauge/tape assembly so the gauge is over previously marked layout line. Gently apply the assembly onto surface. If the assembly is misaligned, lift the tape again at a shallow angle until the assembly is free from the surface. Reposition.

- 7) Lift tape: prior to applying adhesive, lift the end of tape opposite the solder tabs at a shallow angle, until the gauge and terminal is free from the surface. Tack the loose end of the tape under and press to the surface, so the gage lies flat with the bonding side exposed.

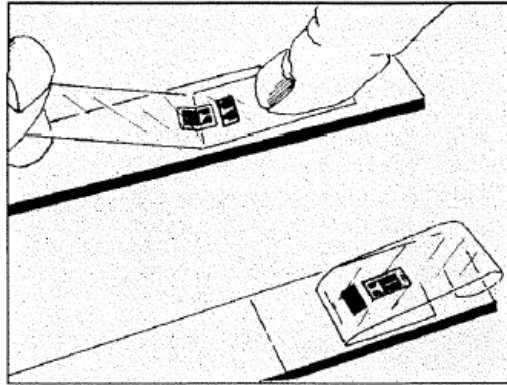


Figure 21 Lift tape

- 8) Apply adhesive and attach: apply a drop of adhesive to the gage's bonding side, attach the gauge and the surface by pressing on the tape for a minute. Wait two minutes before making a firm wiping stroke over the tape.
- 9) Remove the tape and clean the terminals with alcohol and a cotton swab.
- 10) Soldering and stress relief: mask the gage grid area with drafting tape before soldering. After soldering the wires to the terminals, tape or hot gluing the lead-in wires to the surface to prevent the wires from being accidentally pulled from the tabs.

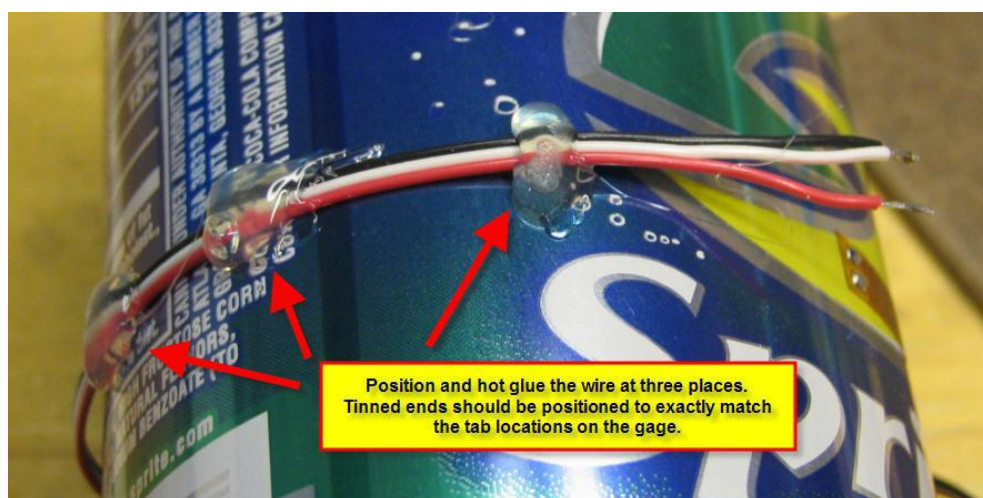


Figure 22 hot gluing the positioned wires at 3 locations

- 11) Protecting the gage: apply a protective coating over the entire gage and terminal area.
- 12) Measure the base resistance of the unstrained strain gage after its proper mounting but before complete wiring. Check for surface contamination by measuring the isolation resistance between the gauge grid and the stressed force detector specimen by means of an ohmmeter, if the specimen is conductive. This should be done before connecting the lead wires to the instrumentation.
- 13)
- 14) Strain gage will be connected to a Wheatstone bridge with quarter bridge set-up.
- 15) Connect the signal conditioner properly to provide power to the bridge and amplify the signal. For set-up procedures, refer to Document 2.
- 16) Connect the inputs from the signal conditioner to the NI DAQ device with a BNC cable, use channel AI0.

3.2.4 Construct the LabVIEW program

Refer to Document 3 for the tutorial to construct a basic VI program for this laboratory.

3.2.5 Verify the Set-up

Before starting the measurements, the strain gauge installations needs to be verified, the following steps should be followed:

- a. Run the VI program to monitor the readings.
- b. Check for irrelevant induced voltages in the circuit by reading the voltage when the power supply to the bridge is disconnected. Ensure that bridge output voltage readings for each strain-gage channel are practically zero.
- c. Connect the excitation power supply to the bridge and verify both the correct voltage level and its stability.
- d. Test out the strain gage bond by applying pressure to the gage. The reading should not be affected.

3.3 Taking Measurements

- a. Before connecting the can, measure the can diameter. Record uncertainty. The middle part of the can has the largest diameter, make sure to capture the diameter from the middle.

- b.** Enter relevant information into VI's front panel, use standard thickness obtain from research as initial value. Press the can or slightly shake the can and observe the measured strains react as expected. Let the can settle (strains and pressure approach zero) before starting measurement.
- c.** Run the VI for 10 seconds, then open the can, keep recording for another 10 to 20 seconds. Press "stop" button to make sure the data is recorded. Data will be saved in csv file in the same directory VI is saved.
- d.** Measure thickness of the can after drinking the beverage; adjust the calculation results if there is significant deviation from the initial value used. The middle part of the can has the lowest thickness, make sure to capture the thickness from the middle.

4. DATA ANALYSIS & DISCUSSION

With the results acquired with three soda cans, estimate the range of internal pressure of similar soda cans. Compare with the pressure value obtained through research.

Conduct uncertainty analysis on the pressure measurements and Poisson's ratio measurements. Assume 3% of uncertainty in strain measurements. Refer to provided sample uncertainty analysis.

Identify, in order of importance, percentage contribution of all uncertainties to the overall uncertainty in pressure characterizations and Poisson's ratio measurements.

*For optional activities during this laboratory, refer to Document 4.

ATTACHMENTS

- Suggested VI
- Sample Lab Report
- User Manual of Signal Conditioner Used in the Experiment

Document 1: Soda Can Parameters and Uncertainty Estimation as a Reference

a. Standard dimension of the soda can (diameter and thickness) and the uncertainties associated

The standard values and factors contributing to uncertainty for can diameter and thickness of a soda can are listed in table below. ²

	Standard		Resolution	Repeatability (Assumed)	Standard Deviation (Assumed)
	Metric	English			
Diameter	6.6 cm	2.6 in	0.001 in	0.005 in	0.0001in
Thickness	0.013cm	0.005in	0.0001 in	0.0001in	negligible

Table 1 Standard values and factors contributing to uncertainty for can diameter and thickness

According to the table above, for diameter, the uncertainty caused by resolution is $(u_{res})_d = 0.0005$ in, the uncertainty caused by repeatability is $(u_{rep})_d = 0.0025$ in, uncertainty caused by standard deviation is $(u_{sd})_d = 0.0001$ in. Therefore, uncertainty in diameter is calculated as:

$$(u_d)_d = \sqrt{(u_{res})_d^2 + (u_{rep})_d^2 + (u_{sd})_d^2} = 0.0025 \text{ in} \quad \text{Eq.14}$$

Similarly, for thickness, the uncertainty caused by resolution is $(u_{res})_t = 0.0005$ in, the uncertainty caused by repeatability is $(u_{rep})_t = 0.0005$ in. Therefore, uncertainty in thickness is calculated as:

$$(u_d)_t = \sqrt{(u_{res})_t^2 + (u_{rep})_t^2} = 0.00071 \text{ in} \quad \text{Eq.15}$$

Same approach can be applied to uncertainty of strain measurement.

Resolution of strain measurements can be extracted by plotting the measurements data points and identifying the smallest increment.

When gain=192, resolution is $0.325 \mu\epsilon$ and uncertainty caused by resolution is $0.325 \mu\epsilon$. Therefore, repeatability is $0.163 \mu\epsilon$, and uncertainty caused by resolution is $0.163 \mu\epsilon$. The strain gauge chosen for this experiment has a gauge factor of $2.095 \pm 0.5\%$. Therefore, when measuring strain of $1000 \mu\epsilon$, the uncertainty caused by gauge factor is $5 \mu\epsilon$.

Eq.16

² Berlage, R. (2001). *Strain Measurements of a Soda Can*. Northern Illinois University.

$$(u_d)_\varepsilon = \sqrt{(u_{res})_\varepsilon^2 + (u_{rep})_\varepsilon^2 + (u_{gf})_\varepsilon^2} = 5.0132 \mu\varepsilon$$

The beverage can lids are usually made from AA5182 H48, while bodies are usually made from AA 3004 or AA 3104 in the H 19 temper. This specification is sufficiently wide to permit suppliers to offer versions with higher formability or higher strength properties. Increase in material strength has been achieved by gradually increasing the magnesium content from the nominal 0.9% of 10 years ago, to nominal 1.1% today, and copper from nominal 0.06 to 0.15%. In addition to magnesium and copper, manganese (up to 1.5%) is the principal alloying element for increasing strength. Careful control of the level of iron, and the iron/silicon ratio, assists in reducing the level of earing; it also assists in controlling the grain size, which is beneficial to formability. 3

b. Material property of soda cans (material type, elastic modulus, and Poisson's ratio) and uncertainty associated.

AA3004-H19's mechanical properties and significant digits of the data are shown in table below.

Mechanical Properties	Metric		English	
	Value	Significant Digits	Value	Significant Digits
Modulus of Elasticity	69.0 GPa	3	10000ksi	3
Poisson's Ratio	0.35	2	0.35	2

Table 2 AA3004-H19's mechanical properties

In Aluminum alloys, the compressive modulus is typically 2% greater than the tensile modulus. Provided value is an average of modulus of elasticity during tension and compression.⁴ Therefore, since strain gauge is in tension during our experiment, the corrected modulus of elasticity should be 68.3 GPa and 9900ksi, uncertainty of modulus of elasticity should be approximately 1%. And uncertainty of Poisson' ratio is about 3%.

³ Wootton, E. (n.d.). *Case study on Can Making*. Retrieved 2012, from Training in Aluminum Application Technologies: <http://core.materials.ac.uk/repository/aaa/talat/3710.pdf>

⁴ Ibid.

c. Common internal pressure range of soda cans

Gases exert a pressure on any surface with which they are in contact. The amount of pressure exerted by the molecules of a gas depends on the force and frequency of the molecules towards the walls of its container. The pressure of gases is therefore dependent upon temperature and volume. The Third Gas Law states that when the volume of a fixed mass of gas is maintained constant, pressure is directly proportional to absolute temperature.

Soda manufacturers often inject cold liquid with pressurized carbon dioxide, then bottle the drink under high pressure. This is due to the fact that more gas will dissolve in a cold liquid that's under a high pressure than in a warm liquid that is not under pressure. The carbon dioxide in the soda forms carbonic acid, which alleviates the sweet taste of the drink. The carbon dioxide also serves an anti-microbiological purpose.

Each type of soda drink contains a different amount of carbon dioxide, and thus has varying amounts of pressure. On average, the 12 ounce soda cans sold in the US tend to have a pressure of roughly 120 kPa (17psi) when canned at 4 °C, and 250 kPa (36 psi) when stored at 20 °C.

Specifically, a refrigerated can of 7UP® contains 210 kPa (31 psi) of pressure. On the other hand, Pepsi-Cola® contains 276 kPa (40 psi) at approximately 16 °C. Lastly, a can of Coca-Cola Classic® at 34 °C has an internal pressure of approximately 380 kPa (55 psi).⁵

⁵ *Pressure in a can of soda.* (n.d.). Retrieved Dec 2012, from The physics fact book:
<http://hypertextbook.com/facts/2000/SeemaMeraj.shtml>

Document 2: Set-Up Procedure for Signal Conditioner (Tacuna)

a. Connection

Connect the wires as indicated in Figure 1.

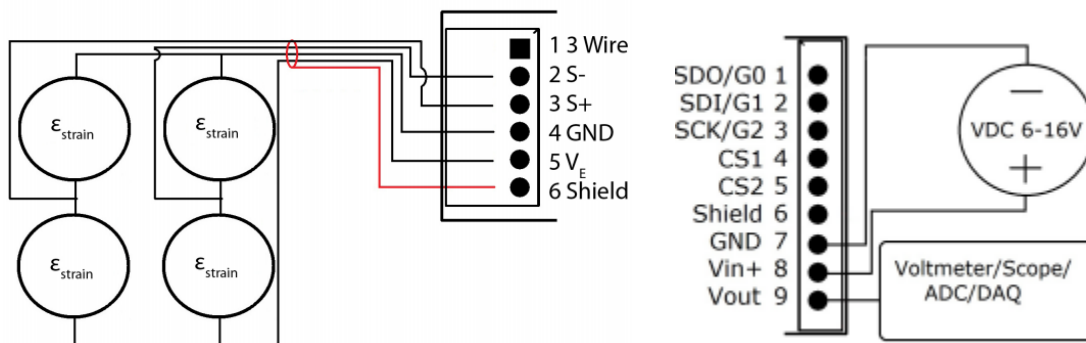


Figure 1 Connections for Tacuna Systems Strain Gauge or Load Cell Amplifier/Conditioner Interface Manual

b. Gain Setting

To get a gain of 220, make sure the switches (location shown in Figure 4) are set as indicated in Table 1.

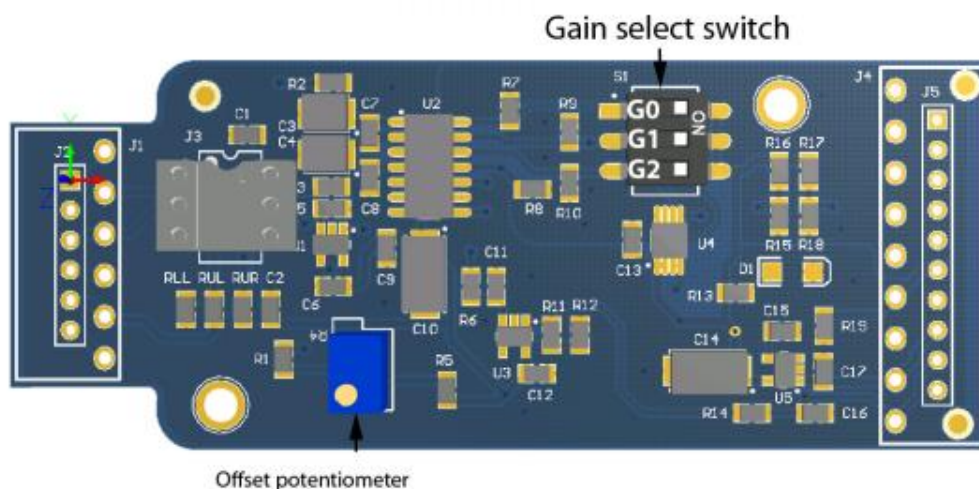


Figure 23 Location of Gain select switch and offset potentiometer

G0	G1	G2
ON	OFF	OFF

Table 9 Switch settings for Tacuna for 220 Gain

c. Bridge Balance

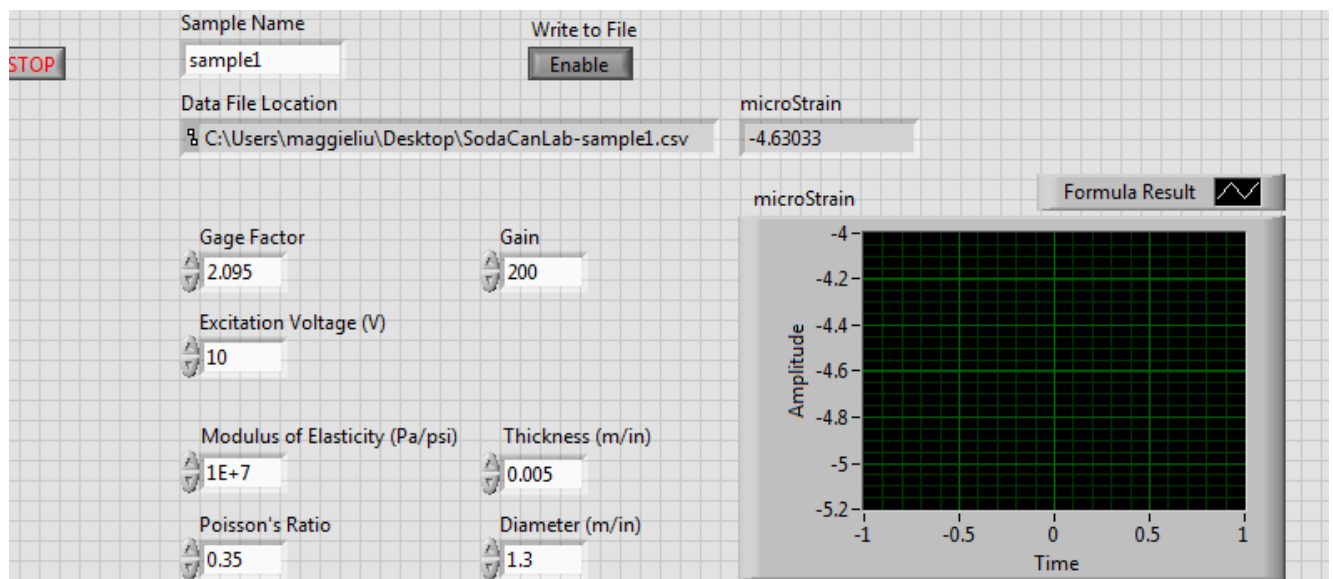
Use the offset potentiometer to adjust the output voltage to 2.5V, which is half of the output range.

It is required to open the enclosure to adjust the gain switches but not the offset potentiometer. The wire connections are located outside of the enclosure.

Document 3: Tutorial for LabVIEW Program

This sample LabVIEW program for the Strain and Pressure Laboratory acquires the voltage input from connected NI DAQ device, calculates and indicates real-time strains experienced by the strain gauge, then calculates and saves dynamic values of internal pressure of the can, stress in both circumferential and axial directions of the can to a .csv file in the same folder where the LabVIEW program is saved, along with the micro strain readings. Around 30 sets of readings are taken each second.

The front panel of the program is shown below. The block diagram is shown on page 2. This document walks through the steps of constructing this program.

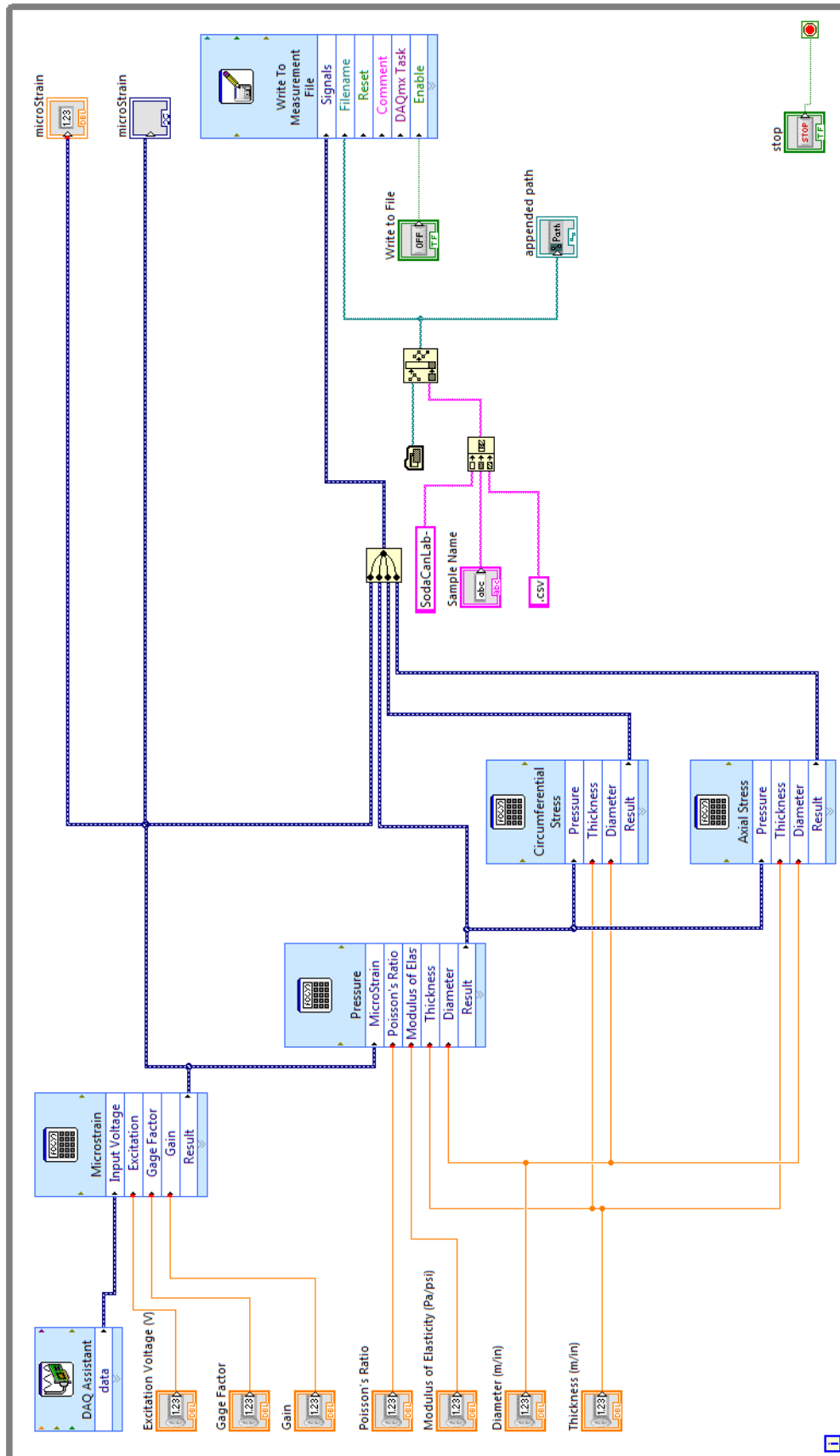


Part 1: Building the Front Panel

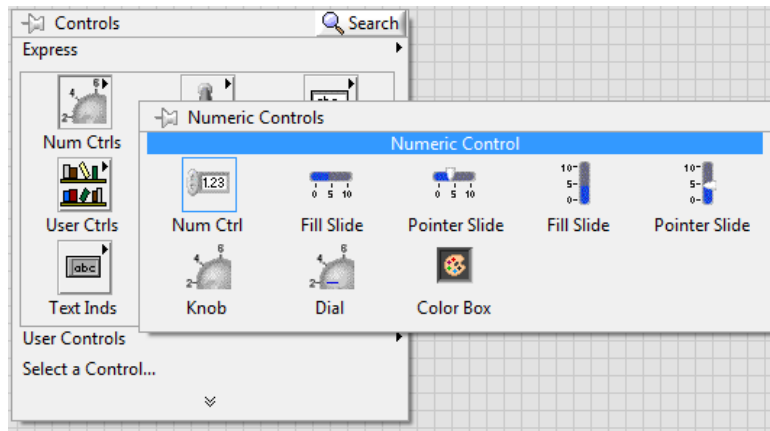
Before opening LabVIEW program, make sure that the NI DAQ device is probably connected to the desktop and turned on.

On *Tools Palette*, make sure that *Automatic Tools Selection* is enabled (the box/button on top of the palette). This setting automatically selects the appropriate pointer tools from the palette based on the mouse- over object.

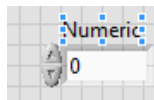




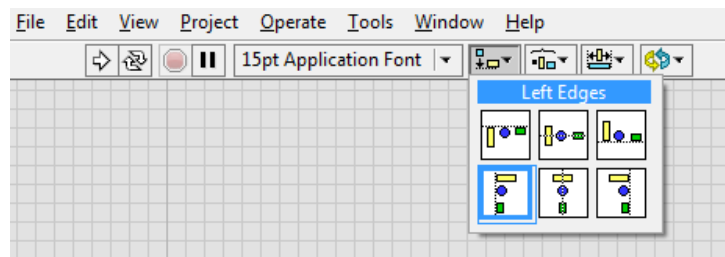
On *Front Panel*, right click on a blank location and access the *Controls Palette*, under *Express* menu find *Numeric Controls*, then select a *Num Ctrl* by left clicking. The control can also be found through *Search* tool in the *Controls Palette*. After selecting the icon, move the pointer to desired location and left click to position the control on front panel.



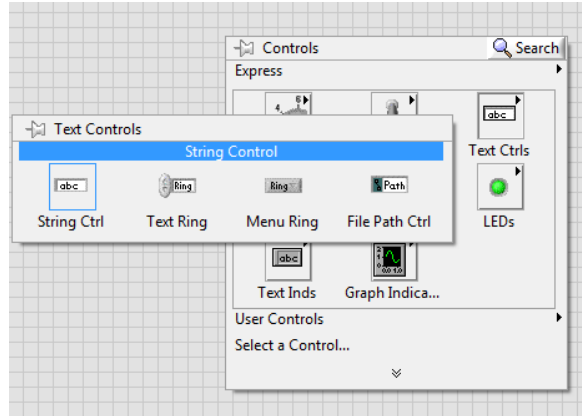
Then, click on the text above the control and edit the name of the control.



In the same way, create all the numeric controls needed for this program. The objects can be arranged with the tools on the top tool bar, alignment, distribution and resizing tools can be used on selected objects.



Add the String Control for Sample Name. The control is located at *Control Palette* → *Express* → *Text Controls* → *String Control*.



Add *Numeric Indicator* and *Waveform Graph* for Micro Strain readings. The path for *Numeric Indicator* is *Control Palette* → *Express* → *Numeric Indicators* → *Numeric Indicator*. The path for *Waveform Graph* is *Control Palette* → *Express* → *Graph Indicators* → *Graph*.

Add *Path Indicator* for Data File Location (*Control Palette* → *Express* → *Text Indicators* → *File Path Indicator*), *Text Button* for Enable Write to File (*Control Palette* → *Express* → *Buttons* → *Text Button*), and *Stop Button* to end the program (*Control Palette* → *Express* → *Buttons* → *Stop Button*).

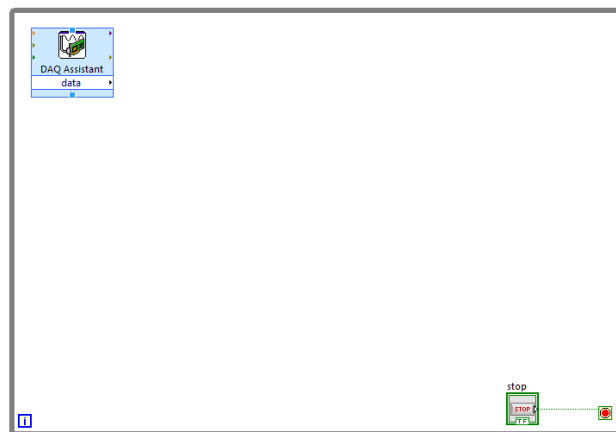
After arranging the objects for a desirable layout, the front panel is completed.

Part 2: Building the Block Diagram

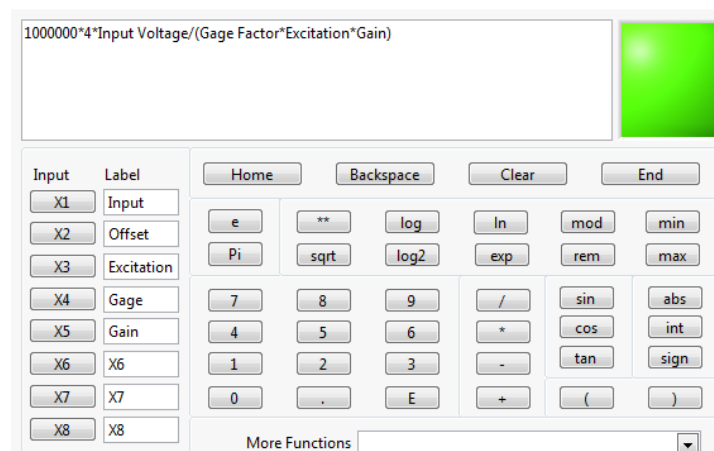
Add a *While Loop* and connect the (already created) *Stop Button* with the *Loop Condition* icon. (*Functions Palette* → *Programming* → *Structures* → *While Loop*).



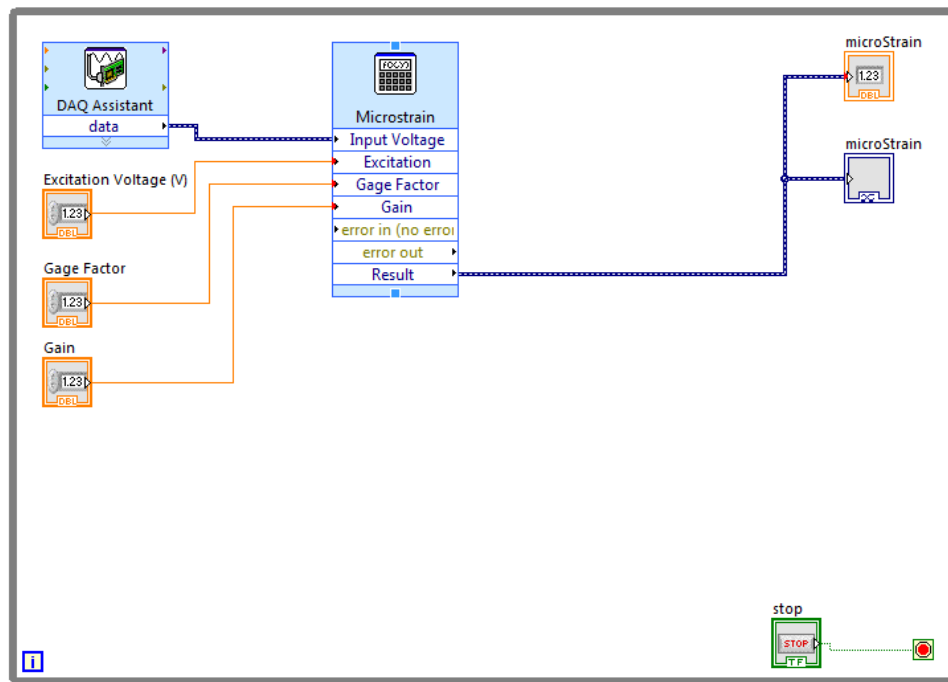
Add a *DAQ Assistant* in the *While Loop* and configure the subVI with the wizard. (*Functions Palette* → *Measurement I/O* → *NI DAQ mx* → *DAQ Assistant*). For the measurement type, select *Acquire Signals* → *Analog Input* → *Voltage*. For the physical channel, select the channel of incoming signal. Since channel AI0 of NI 6229 is connected to the input, select this specific channel. Next, configure the channel settings: input -10V to 10V for input signal range, and *1 Sample (On Demand)* for acquisition mode. Note that the DAQ box needs to be connected to the computer and turned on before starting of LabVIEW program. Save the work and restart the program if the module fails to initialize.



The next step is to transfer the input voltage signal to strain, in the unit of micro strain, and then display it in numerical and graphical form. Create a *Formula* (*Function Palette* → *Express* → *Arithmetic & Comparison* → *Formula*), set “Input Voltage”, “Excitation”, “Gage Factor” and “Gain” as input labels, then click “OK” to close the configuration wizard. Input the formula as shown in the figure below.



On the block diagram, drag down the arrow on the bottom of the *Formula* icon to expand the input/output menu. To change the order of the elements, right click on an element and select “select input/output”, then click on the input/output desired for the position. Connect the *data* output of the *DAQ Assistant*, and the *Numerical Controls* for excitation voltage, gage factor and gain to the corresponding inputs of the *Formula*. Connect the *Numeric Control* and *Wave Graph* for micro strain to the *result* output of *Formula*. Then click on the text under the icon to change the label to “micro strain”.



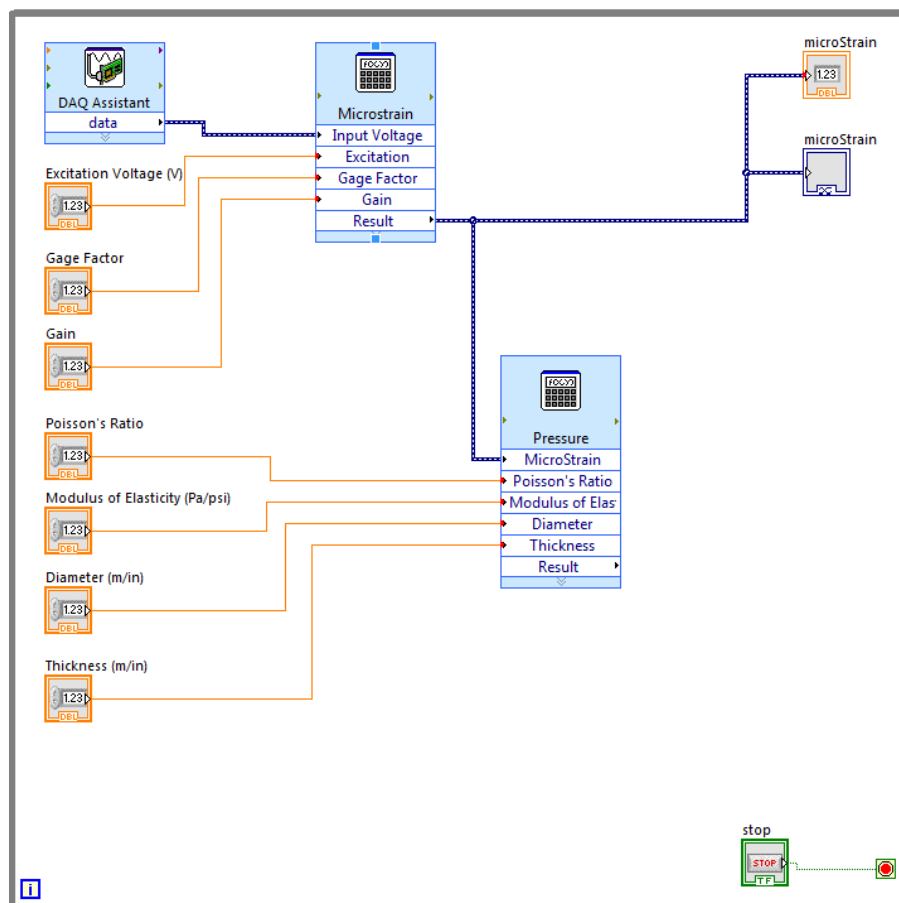
In similar ways, internal pressure can be calculated from strain and other properties of the can. Use *Formula* to calculate internal pressure based on micro strain results from previous *Formula*, inputs from *Numeric Controls* for material’s modulus of elasticity, material’s Poisson’s ratio, thickness of the can wall, and diameter of the can. The formula is shown in the figure below.

Modulus of Elasticity*Thickness*(MicroStrain/1000000)/(Diameter/2*(1-Poisson's Ratio/2))

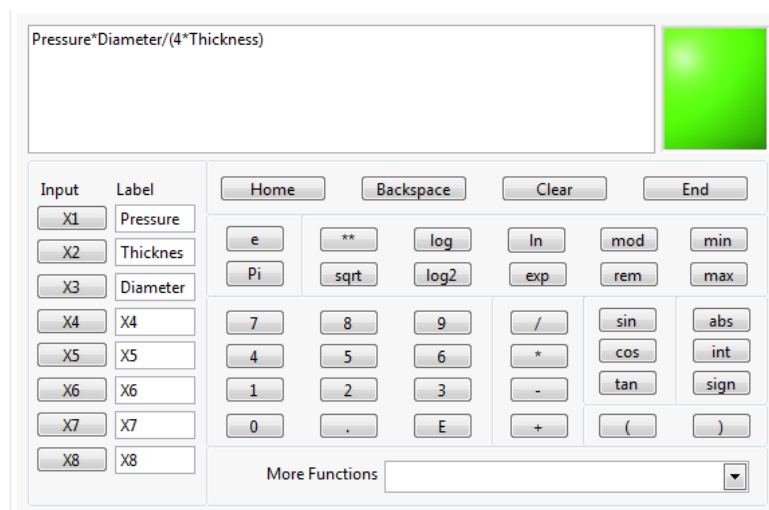
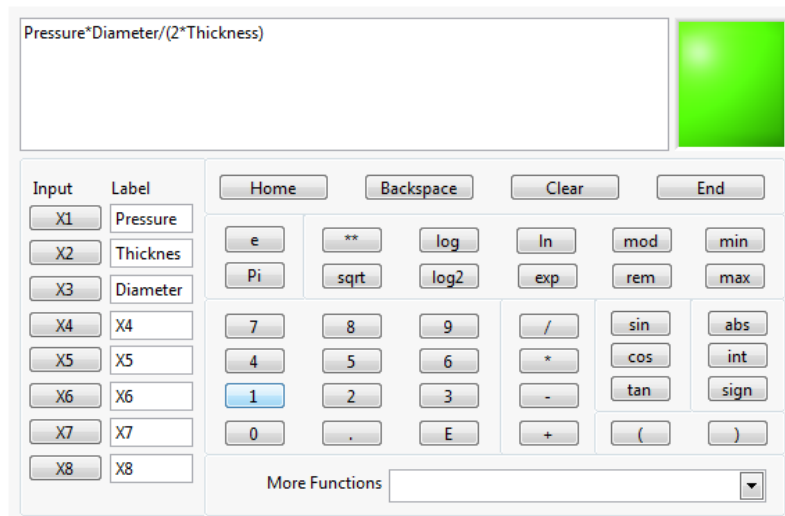
Input	Label
X1	Modulus
X2	Poisson's
X3	Thicknes
X4	Diameter
X5	MicroStra
X6	X6
X7	X7
X8	X8

Home		Backspace		Clear		End	
e	**	log	ln	mod	min		
Pi	sqrt	log2	exp	rem	max		
7	8	9	/	sin	abs		
4	5	6	*	cos	int		
1	2	3	-	tan	sign		
0	.	E	+	(	)		

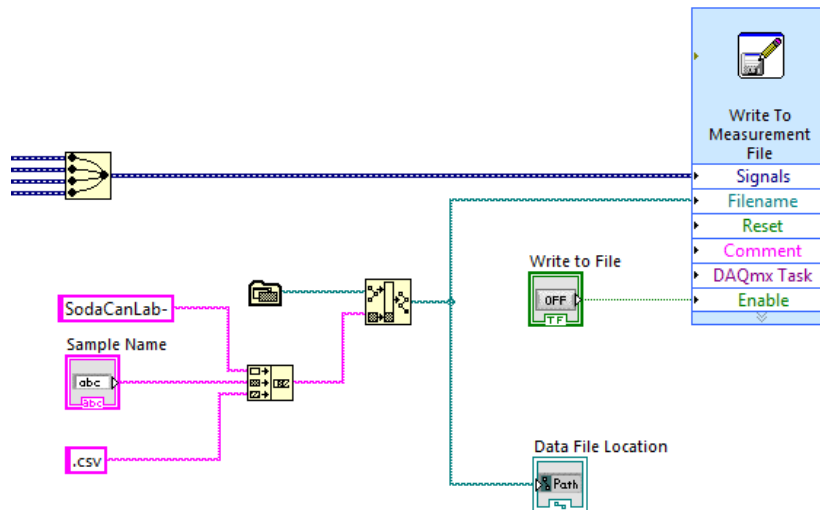
More Functions



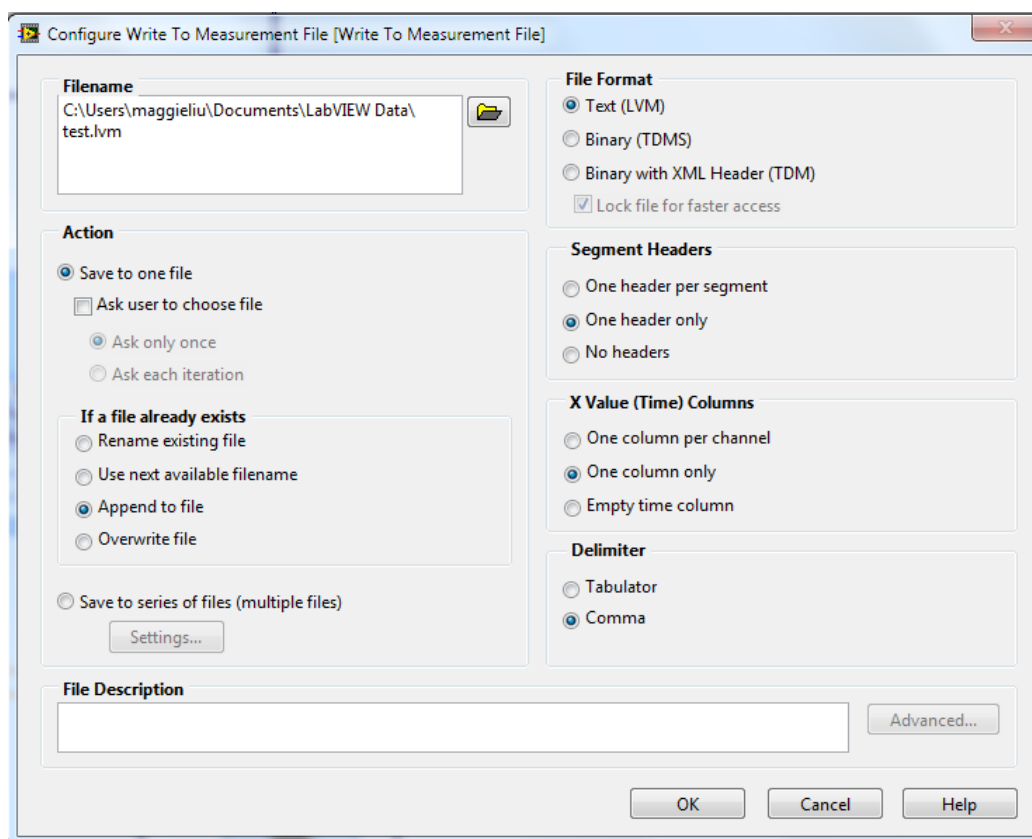
Calculated pressure, diameter and thickness are used to calculate circumferential stress and axial stress. The formulas are shown in the two figures below.



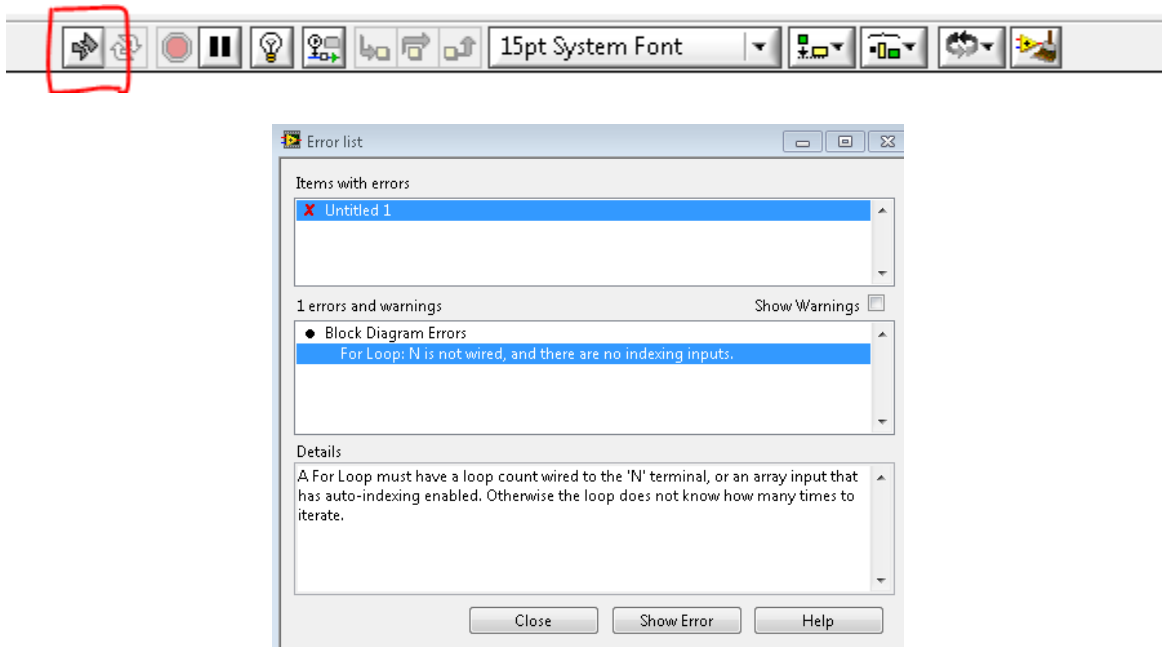
Dynamic data of micro strain, pressure, circumferential and axial stress are then combined with *Merge Signal* function and then written to file with a *Write to Measurement File* function. The merged signal should be connected to *Signal* Input of the *Write to Measurement File* function. The *Filename* can be constructed with *Build Path* function. It builds the file path with an *Application Directory* function, which points to the folder where the VI is saved, and a concatenated string (Use *Concatenate String* function in *String Palette*) which consists of the lab name, the user inputted sample name, and a “.csv” (comma separated values) as file extension, so that the data file can be opened with Microsoft Excel.



The *Write to Measurement File* should be configured as shown below. The filename in this wizard will be overwritten by the input; it should “save to one file”; the format should be text, with one header only or no headers; there should be only one time column; and the delimiter should be comma.



Now we have completed constructing the VI. If there is any error in the program, the run button will appear “broken” as shown in the figure below. Click on the button to view the error list, the “details” should explain the error. Debug until all errors are resolved; use other debugging functions on the menu bar if needed.



When the run button appears as a rightward arrow, enter appropriate parameters on the *Front Panel*, connect a BNC cable to AI0 of the DAQ device with two idle clips (this will provide some varied voltage inputs), and test run the program. Use *Edit* ➔ *Make current values default* to save the entered parameters as default values. If there is no error interrupting the run, we can check the data file under the specified directory for satisfactory results. Trouble shoots until the program is ready for use.

Now the VI is ready for the Soda Can Laboratory. Can you make it better?

Document 4: Optional Activities in this Laboratory

1. Create a shared data file for the class; consolidate measured internal pressure from all the students. What is the average and standard deviation of the measured value? What are some of the possible causes of these variations?
2. Before opening the can in this experiment, shake the can for 5 seconds, measure the change in internal pressure. What are the possible causes of the change?
3. Before opening the can, take two data recordings, one with the DAQ Assistants' input voltage range set to -10V to 10V, one with it set to -2V to 2V. Analyze the data and find out the resolution of each recording. Why are they different?
4. Read the user manual for the signal conditioner and change the gain setting. Compare the resolutions of strain readings and pressure readings under different gains.

Appendix 4: Instructions for Vibration Measurement Laboratory

Laboratory: Vibration Measurements

1. OBJECTIVES

This laboratory uses strain gauge to measure the dynamic characteristic and the elastic material properties of a cantilever.

Vibration data will be analyzed to:

- Determine the vibration amplitude, velocity, and acceleration in various units of measure;
- Determine natural frequencies;
- Measure and express damping characteristics as logarithmic decrement and percentage of critical damping;
- Compare measurements with analytical and/or computational models of a cantilever; and
- Determine elastic modulus of a cantilever.

Uncertainty analysis of the results will be performed.

2. BACKGROUND

Health monitoring is the process of studying and assessing the integrity of structures, which is crucial for preventing failure and for achieving reliable designs. Health monitoring can be done by dynamic or static analysis, or a combination of both. In static analysis, deformations or changes in the orientation of structures, due to application of loads, or unexpected damages, are determined via comparisons with reference models. For dynamic analysis, dynamic characteristics of the structures, including natural frequencies, modal shapes, and damping factors, are determined via modal analysis.

In either static or dynamic health monitoring, the utilization of appropriate transducers is required to provide accurate measurement of structural responses in both frequency and time domains. Conventional devices utilized for health monitoring are based on piezoelectric transducers. These transducers are usually large in size, require high actuation power, and have narrow frequency bandwidths, which reduce their accuracy, versatility, and applicability to study smaller structures. The advanced developments of IC microfabrication and microelectromechanical systems (MEMS) have led to the progressive designs of small footprint, low dynamic mass and actuation power MEMS inertial sensors. Due to their high natural frequencies, these MEMS inertial sensors provide wide frequency bandwidths and high measuring accuracies.

2.1 Static Analysis of a Simple Cantilever Beam

2.1.1 Stress, Strain, and Deflection Associated with Bending

A bending moment exists in a structural element when a moment is applied so that the element bends. The bending moment at a section of a structural element is defined as the sum of the moments about that the section of external forces acting to one side of the section. Moments are calculated by multiplying the external vector forces by the vector distance at which they are applied.

Bending occurs locally when a slender object is subjected to an external load applied perpendicular to a longitudinal axis of the object. On a bending beam, compressive and tensile forces develop in the direction of the beam axis under bending loads. The forces induce stresses on the beam. The maximum compressive force occurs on at the lower edge of the beam, and the

maximum tensile force occurs at the upper most edge. The equation for determining the bending stress is

$$\sigma = \frac{Mc}{I} \quad \text{Eq.1}$$

where M is the applied moment, c is the distance from the neutral axis to the outer fiber of the beam, and I is the moment of inertia. The derivation of Eq.1 is shown in Appendix A.

The maximum bending stress in a beam is

$$\sigma_{max} = \frac{Mt}{2I} \quad \text{Eq.2}$$

where t is the thickness of the beam.

Hooke's law describes the relationship between stress and induced strains for linear elastic materials.

$$\varepsilon = \frac{\sigma}{E} \quad \text{Eq.3}$$

where E is the elastic modulus of the beam's material.

Deflection is the degree to which a structural element is displaced under a load; it may refer to an angle or a distance. As shown in Figure 24, on the neutral axis of a beam subjected to bending, for a very small angle, slope of the beam $\frac{dw}{dx} = \tan \theta = \theta$. The curvature of a beam is defined as $\kappa = \frac{1}{\rho} = \frac{d\theta}{ds}$; ρ is the radius of the curve. Since $d\theta$ is small, $dx = ds$. Therefore we have

$$\kappa = \frac{1}{\rho} = \frac{d\theta}{ds} = \frac{d\theta}{dx} = \frac{d}{dx} \left(\frac{dw}{dx} \right) = \frac{d^2w}{dx^2} \quad \text{Eq.4}$$

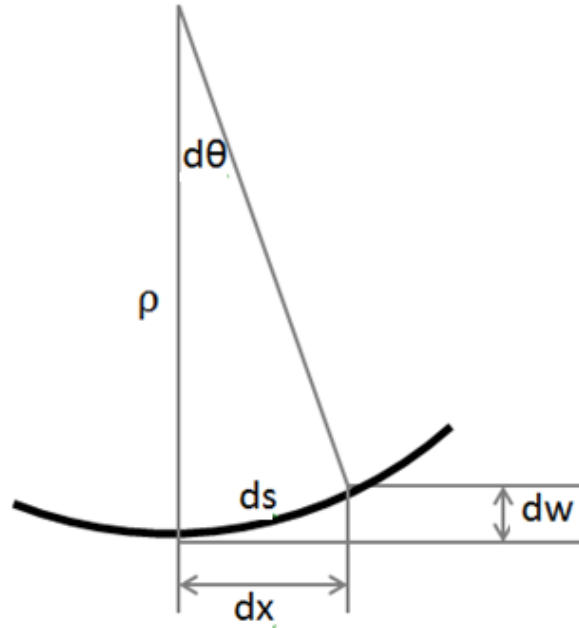


Figure 24 Neutral axis of a beam subjected to bending

Euler—Bernoulli beam theory relates curvature of a bending beam to bending moment and rigidity of the material

$$EI \frac{d^2w}{dx^2} = M \quad \text{Eq.4}$$

where E is the elastic modulus of the material, I is the second moment of area. I must be calculated with respect to the centroidal axis perpendicular to the applied loading. w is the deflection in distance, $\frac{dw}{dx}$ is the slope of the beam, and $\frac{d^2w}{dx^2}$ equals to the beam curvature κ , or $\frac{1}{\rho}$. The second moment of inertia of rectangle about the centroidal axis perpendicular to the applied loading is expressed as

$$I = \frac{bT^3}{12} \quad \text{Eq.5}$$

where b is the width and T is the height or thickness.

2.1.2 Calculation of Static Characteristics

Macaulay's method (the double integration method) is a technique used in structural analysis to determine the deflection of Euler-Bernoulli beams. Use of Macaulay's technique is very convenient for cases of discontinuous and/or discrete loading. Typically partial uniformly

distributed loads and uniformly varying loads over the span and a number of concentrated loads are conveniently handled using this technique.

For general loadings, the bending moment M can be expressed in the form

$$M = M(x) + P_1\langle x - a_1 \rangle + P_2\langle x - a_2 \rangle + \cdots + P_n\langle x - a_n \rangle \quad \text{Eq.6}$$

The quantity $\langle x - a_i \rangle$ is a Macaulay bracket, it is defined as

$$\langle x - a_i \rangle = \begin{cases} 0 & \text{if } x < a_i \\ x - a_i & \text{if } x > a_i \end{cases} \quad \text{Eq.7}$$

When integrating expressions containing Macaulay brackets, we have

$$\int P\langle x - a \rangle dx = \frac{P\langle x - a \rangle^2}{2} + C_m \quad \text{Eq.8}$$

Consider a simple cantilever beam fixed at one end and loaded with a force on the free end. The dimensions of the cantilever beam are defined in the figure below.

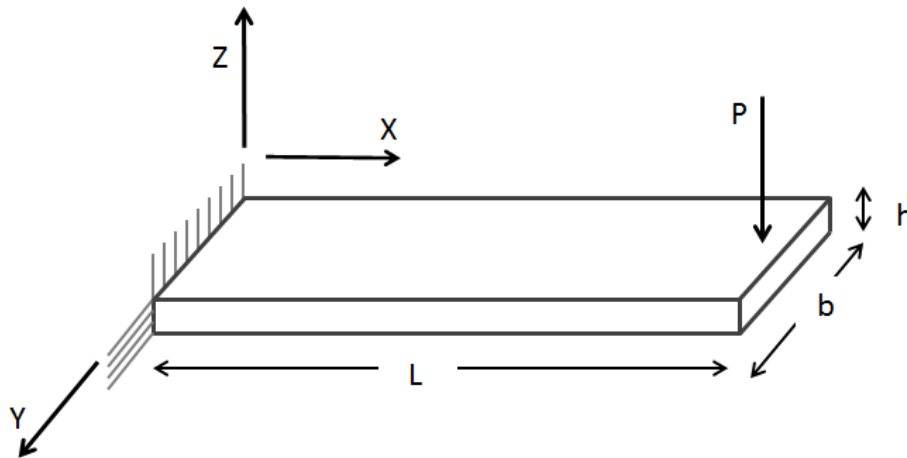


Figure 25 Dimensions of a Simple Cantilever Beam

Figure 3 equations and plots for deflection in terms of distance, deflection in terms of slope, bending moment and shear stress at arbitrary location in the beam on the neutral axis. In the equations, x is the distance from the fixed end of the beam to the point of interest, P is the applied load, L is the length of the beam, E is the elastic modulus, and I is the second moment of inertia.

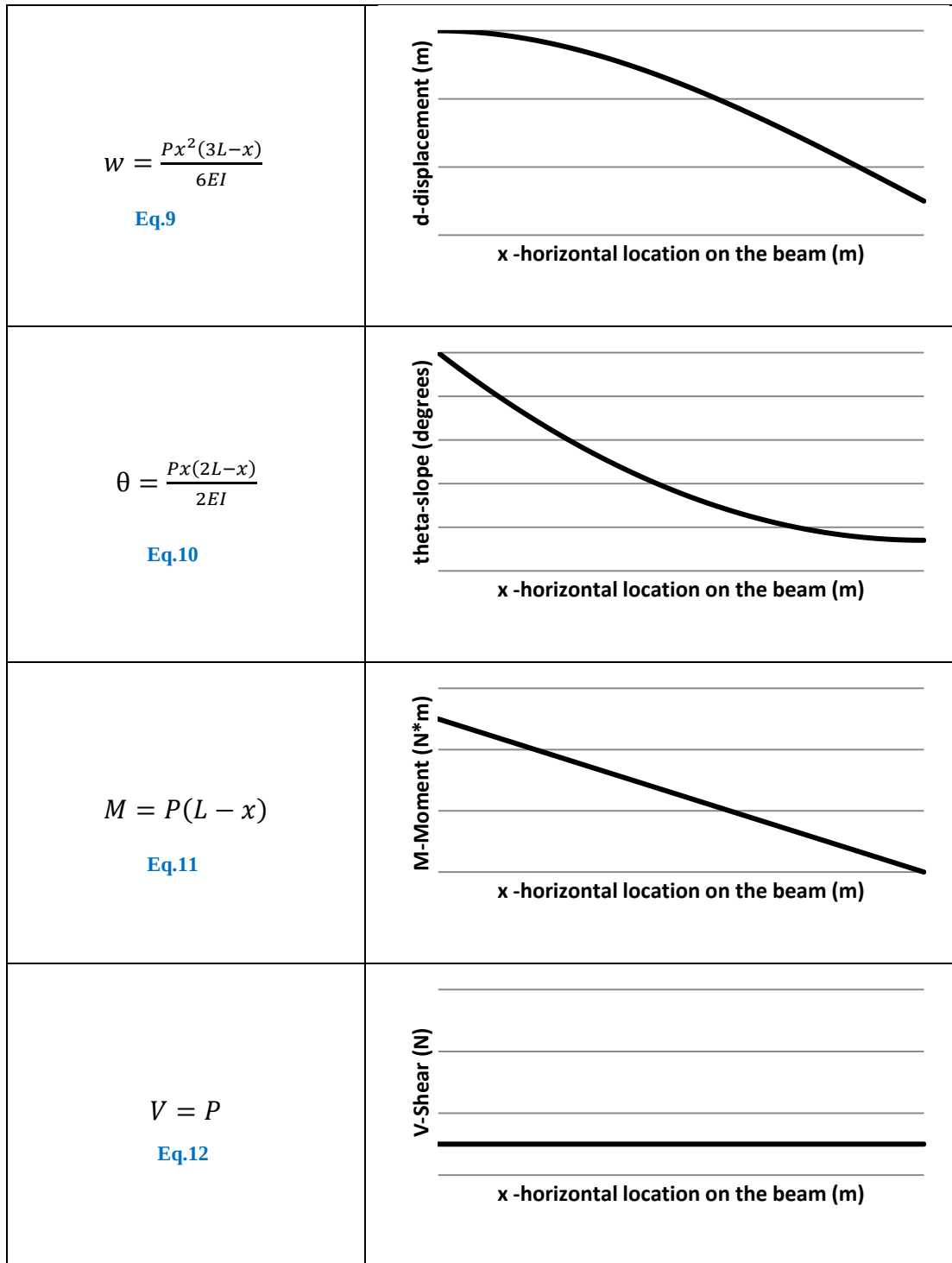


Figure 3 Deflection, Bending Moment and Shear Stress

Recall Eq.2 and Eq.3, and substitute with $M = P(L - x)$, we have the expressions for maximum bending stress and corresponding strain at arbitrary location on beam.

$$\sigma_{max} = \frac{P(L-x)t}{2I} = \frac{6P(L-x)}{bt^2} \quad \text{Eq.13}$$

$$\varepsilon = \frac{P(L-x)t}{2EI} = \frac{6P(L-x)}{EbT^2} \quad \text{Eq.14}$$

2.2 Dynamic Characteristics of a Cantilever Beam under Free Vibration

Vibration is a mechanical phenomenon whereby oscillations occur about an equilibrium point. Free vibration occurs when a mechanical system is set off with an initial input and then allowed to vibrate freely. The mechanical system will then vibrate at one or more of its "natural frequency" and damp down to zero. Forced vibration is when an alternating force or motion is applied to a mechanical system.

A normal mode of an oscillating system is a pattern of motion in which all parts of the system move sinusoidally with the same frequency and with a fixed phase relation. The motion described by the normal modes is called resonance. The frequencies of the normal modes of a system are known as its natural frequencies or resonant frequencies. Each physical object has a set of normal modes that depend on its structure, materials and boundary conditions.

A mode of vibration is characterized by a modal frequency and a mode shape, and is numbered according to the number of half waves in the vibration. In a system with two or more dimensions, such as the pictured disk, each dimension is given a mode number. Each mode is entirely independent of all other modes. Thus all modes have different frequencies (with lower modes having lower frequencies) and different mode shapes.

2.2.1 Natural Frequencies of a Cantilever Beam under Free Vibration ⁶

For an Euler-Bernoulli beam under free vibration, the Euler-Lagrange equation is

$$EI \frac{d^4 w}{dx^4} + \rho A \frac{d^2 w}{dt^2} = 0 \quad \text{Eq.15}$$

Since deflection is a function of time and distance, we have

$$w(x, t) = W(x)\cos(\omega t - \alpha) \quad \text{Eq.16}$$

This makes eq.15:

⁶ Volterra, E. (01/01/1966). "Dynamics of Vibrations". *Journal of applied mechanics*(0021-8936), 33(4), p.956.

$$\frac{EI}{\rho A} \frac{d^4 w}{dx^4} - \omega^2 w = 0 \quad \text{Eq.17}$$

Solution for displacement is:

$$W(x) = C_1 \cos(x\lambda) + C_2 \sin(x\lambda) + C_3 \cosh(x\lambda) + C_4 \sinh(x\lambda) \quad \text{Eq.18}$$

$$\text{Where: } \lambda = \left(\frac{\rho A}{EI} \omega^2 \right)^{\frac{1}{4}}$$

For a cantilever beam, the displacement and slope are zero at the fixed end, and the moment and shear are zero at the free end. Thus the boundary conditions are:

$$\text{when } x = 0, y = 0, \frac{dy}{dx} = 0.$$

$$\text{when } x=L, \frac{d^2 y}{dx^2} = 0, \frac{d^3 y}{dx^3} = 0.$$

Applying the boundary conditions yields

$$\cos(\lambda L) \cosh(\lambda L) = -1 \quad \text{Eq.19}$$

The equation for time is

$$T(t) = b_1 \sin[(\beta_n^2 \sqrt{\frac{EI}{\rho A}}) t] + b_2 \cos[(\beta_n^2 \sqrt{\frac{EI}{\rho A}}) t] \quad \text{Eq.20}$$

So the exact expression of n^{th} natural frequency in rad/sec is

$$\omega_n = \frac{(\lambda L)^2}{L^2} \sqrt{\frac{EI}{\rho A}} = \frac{\alpha_n^2}{L^2} \sqrt{\frac{EI}{\rho A}} \quad \text{Eq.21}$$

where E is Young's modulus of elasticity, I is moment of inertia of cross section, L is effective length of beam, and ρ is the density, A is the area of cross section. The dimensionless wave number $\beta = 2\pi/\text{wavelength}$. β_n values for cantilever beams are: $\beta_1 L = 1.8751 = \alpha_1$, $\beta_2 L = 4.6941 = \alpha_2$, $\beta_3 L = 7.8548 = \alpha_3$, $\beta_4 L = 10.99557 = \alpha_4$, $\beta_5 L = 14.1372 = \alpha_5$, $\beta_6 L = 17.279 = \alpha_6$.

Therefore, the natural frequency of cantilever beam with a rectangular cross section is

$$\omega_n = \alpha_n^2 \sqrt{\frac{EI}{mL^4}} = \alpha_n^2 \sqrt{\frac{EI}{\rho AL^4}} = \alpha_n^2 \sqrt{\frac{E(\frac{bT^3}{12})}{\rho(bT)L^4}} = \frac{\alpha_n^2}{2\sqrt{3}} \sqrt{\frac{ET^2}{\rho L^4}} \quad \text{Eq.22}$$

A simple method of approximating the natural frequency of cantilever beams is shown below. The method also estimates equivalent stiffness and equivalent mass of the beam.

Recall the generic expression of natural frequency in rad/sec is $\omega_n = \sqrt{\frac{k}{m}}$. To find the natural frequency of a cantilever beam, the equivalent stiffness and equivalent mass are needed.

As given in section 2.1.2, the deflection w at the tip of a cantilever beam ($x=L$) is

$$w = \frac{Px^2(3L-x)}{6EI} = \frac{PL^3}{3EI} \quad \text{Eq.23}$$

Using Hook's law, the deflection at the end of the cantilever can be expressed as

$$P = kw \quad \text{Eq.24}$$

where k is the stiffness of the cantilever beam. Combining eq. 17 and eq. 18, k can be given as

$$k = \frac{3EI}{L^3} \quad \text{Eq.25}$$

Therefore, the frequency of a cantilever with a point load m at length x can be given as

$$\omega_n = \sqrt{\frac{3EI}{mx^3}} \quad \text{Eq.26}$$

The same frequency can be provided by a load m_{eq} at the end of beam

$$\omega_n = \sqrt{\frac{3EI}{m_{eq}L^3}} \quad \text{Eq.27}$$

Consider a cantilever beam with constant cross section and uniformly distributed mass of value m per meter along the length. At any time t during vibration, the relationship between generic deflection (measured at an abscissa y from free end), denoted by $w_y(t)$ and the deflection at the free end, denoted by $w_{max}(t)$ can be expressed as:

$$w_y(t) = \left[1 - \frac{3}{2} \left(\frac{y}{L} \right) + \frac{1}{2} \left(\frac{y}{L} \right)^3 \right] w_{max}(t) \quad \text{Eq.28}$$

The kinetic energy of the distributed parameter cantilever is expressed as:

$$T = \frac{1}{2} \int_0^L \rho A \left[\frac{\partial w_y(t)}{\partial t} \right]^2 dy = \frac{1}{2} \left[\frac{\partial w_{max}(t)}{\partial t} \right]^2 \int_0^L \rho A \left[1 - \frac{3}{2} \left(\frac{y}{L} \right) + \frac{1}{2} \left(\frac{y}{L} \right)^3 \right]^2 dy \quad \text{Eq.29}$$

The lumped load m_{eq} at the end of beam has the kinetic energy:

$$T_c = \frac{1}{2} m_{eq} \left[\frac{dw_{max}(t)}{dt} \right]^2 \quad \text{Eq.30}$$

The two kinetic energies of Eq. 29 and Eq.30 need to be equal. The equivalent mass is:

$$m_{eq} = \int_0^L \rho A \left[1 - \frac{3}{2} \left(\frac{y}{L} \right) + \frac{1}{2} \left(\frac{y}{L} \right)^3 \right]^2 dy = \frac{33}{140} m \quad \text{Eq.31}$$

Therefore, the natural frequency in rad/sec is expressed as:

$$\omega_n = 3.5675 \sqrt{\frac{EI}{mL^3}} \quad \text{Eq.32}$$

The error of the estimation is within 2%.

2.2.2 Mode Shapes of a Cantilever Beam under Free Vibration

The mode shapes of a vibrating beam can be determined through solving the relevant equations. The video below shows the vibration mode shapes of a simply supported beam and a cantilever beam.

<http://www.youtube.com/watch?v=kun62B7VUg8>

2.2.3 Damping Factor of a Cantilever Beam under Free Vibration

The vibrating object dissipates energy through damping, and the oscillation amplitude decays with time as a result. The damping ratio is a dimensionless measure describing how rapidly the oscillations decay during each cycle.

Where the system is completely lossless, the mass would oscillate indefinitely, with constant amplitude. This hypothetical case is called undamped.

If the system contained high losses, for example if the system vibrates in a viscous fluid, the mass could slowly return to its rest position without ever overshooting. This case is called overdamped. Commonly, the mass tends to overshoot its starting position, and then return, overshooting again. With each overshoot, some energy in the system is dissipated, and the oscillations die towards zero. This case is called underdamped. Between the overdamped and

underdamped cases, there exists a certain level of damping at which the system will just fail to overshoot and will not make a single oscillation. This case is called critical damping. The key difference between critical damping and overdamping is that, in critical damping, the system returns to equilibrium in the minimum amount of time.

The damping ratio expresses the level of damping in a system relative to critical damping. For a damped harmonic oscillator with mass m , damping coefficient c , and spring constant k , it can be defined as the ratio of the damping coefficient in the system's differential equation to the critical damping coefficient:

$$\zeta = \frac{c}{c_c} \quad \text{Eq.33}$$

where the system's equation of motion is

$$m \frac{d^2w}{dt^2} + c \frac{dw}{dt} + kw = 0 \quad \text{Eq.34}$$

and the corresponding critical damping coefficient is

$$c_c = 2\sqrt{km} \quad \text{Eq.35}$$

A common method for analyzing the damping of an underdamped oscillation is the logarithmic decrement method, for which the following relationships apply.

$$\ln\left(\frac{y_i}{y_{i+n}}\right) = n\delta \quad \text{Eq.36}$$

$$\zeta = \frac{\delta}{\sqrt{(2\pi)^2 + \delta^2}} \quad \text{Eq.37}$$

$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}} \quad \text{Eq.38}$$

where y_i is the amplitude of peak i (i is an integer counting each peak), n is the number of cycles being considered, δ is the log decrement, ω_n is the undamped natural frequency, and ω_d is the damped natural frequency. Both frequencies are in radian per second. Note, it is assumed that object oscillates about zero. If there is an offset in y , the y_i amplitude must be defined relative to that offset.

According to Eq.25 and Eq.33, the equivalent stiffness and equivalent mass are expressed as:

$$k = \frac{3EI}{L^3}$$

$$m_{eq} = \frac{33}{140} m$$

The critical damping factor of a cantilever beam is

$$c_c = 1.6818 \sqrt{\frac{EI}{L^3}} m = 0.2428 \sqrt{\frac{Eb^3t}{L^3}} m \quad \text{Eq.39}$$

2.3 Measurement Methods of Dynamic Characteristics

The dynamic characteristics of a vibrating object, including vibrating frequency and damping factor extracted from strain and acceleration data acquired during the vibration .

2.3.1 Measurement of Vibration Frequency: Fourier Transformation

Fourier series decomposes periodic signals into the sum of an infinite series of simple oscillating functions, namely sines and cosines, or complex exponentials. The technique can be applied to mathematical and physical problems, especially electrical engineering, vibration analysis, acoustics, optics, signal processing, image processing, quantum mechanics, econometrics, etc.

2.3.1.1 Fourier Series

The Fourier series of a periodic function consists of an offset value, an even (cosine) component, and an odd (sine) component. The offset value a_0 of a periodic function $f(x)$ with period T is defined as the average value of the periodic function over a period. The even component relates to the portion of the periodic function behaving as $f(x) = f(-x)$, which is a property of the cosine function. The odd component relates to the portion of the periodic function behaving as $f(x) = -f(-x)$, which is a property of the sine function.

The components a_0 , a_n , and b_n are given by the relationships

$$a_0 = \frac{1}{T} \int_{-T/2}^{T/2} f(x) dx \quad \text{Eq.40}$$

$$a_n = \frac{2}{T} \int_{-T/2}^{T/2} f(x) \cos(nx) dx \quad \text{Eq.41}$$

$$b_n = \frac{2}{T} \int_{-T/2}^{T/2} f(x) \sin(nx) dx \quad \text{Eq.42}$$

The three components are combined to form the Fourier series:

$$f(x) = a_0 + \sum_{n=1}^{\infty} [a_n \cos(nx) + b_n \sin(nx)] \quad \text{Eq.43}$$

The limit of the Fourier series approaches the exact value of the periodic function as the number of terms in the series approaches infinity. The Fourier series become an approximation when the series includes a finite number of terms. More terms in the series expansion, closer the approximation of the original function, as demonstrated in Figure 4 Fourier series expansion of a periodic sawtooth wave (L=1). The number of terms in the series varies from one, two, to five and 25.

, which contains Fourier series approximations of a saw tooth signal with 1 term, 2 terms, 5 terms and 25 terms.

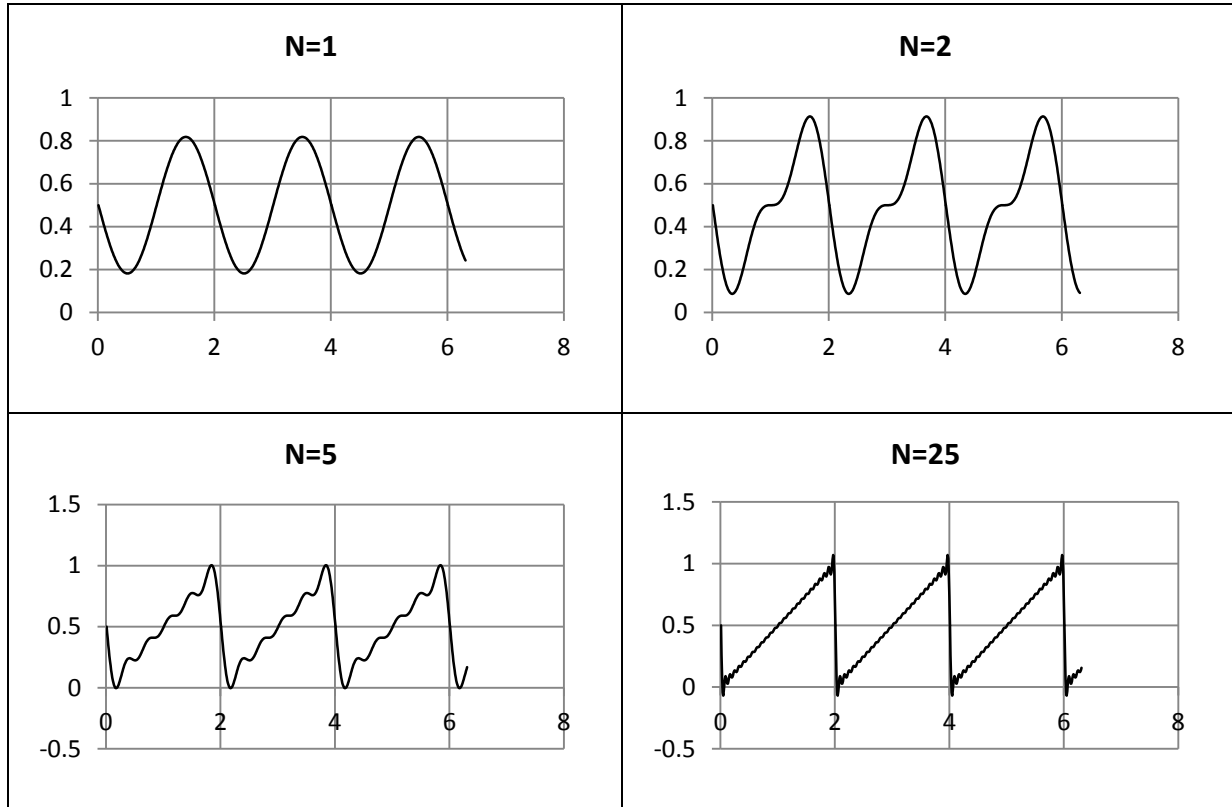


Figure 4 Fourier series expansion of a periodic sawtooth wave (L=1). The number of terms in the series varies from one, two, to five and 25.

The derivation of the Fourier functions for a periodic sawtooth wave is shown below.

Consider a string of length 2L plucked at the right end and fixed at the left. The functional form of this configuration is

$$f(x) = \frac{x}{2L} \quad \text{Eq.44}$$

The components of the Fourier series are given by

$$a_0 = \frac{1}{L} \int_0^{2L} \frac{x}{2L} dx = 1 \quad \text{Eq.45}$$

$$a_n = \frac{1}{L} \int_0^{2L} \frac{x}{2L} \cos\left(\frac{n\pi x}{L}\right) dx = \frac{[2n\pi \cos(n\pi)] \sin(n\pi)}{n^2 \pi^2} = 0 \quad \text{Eq.46}$$

$$b_n = \frac{1}{L} \int_0^{2L} \frac{x}{2L} \sin\left(\frac{n\pi x}{L}\right) dx = \frac{-2n\pi \cos(2n\pi) + \sin(2n\pi)}{2n^2 \pi^2} = -\frac{1}{n\pi} \quad \text{Eq.47}$$

The Fourier series is therefore given by

$$f(x) = \frac{1}{2} - \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{n\pi x}{L}\right) \quad \text{Eq.48}$$

The example of periodic square wave can be also used to illustrate Fourier approximation.

Consider a square wave of length $2L$ over the range $[0, 2L]$. The functional form of the configuration

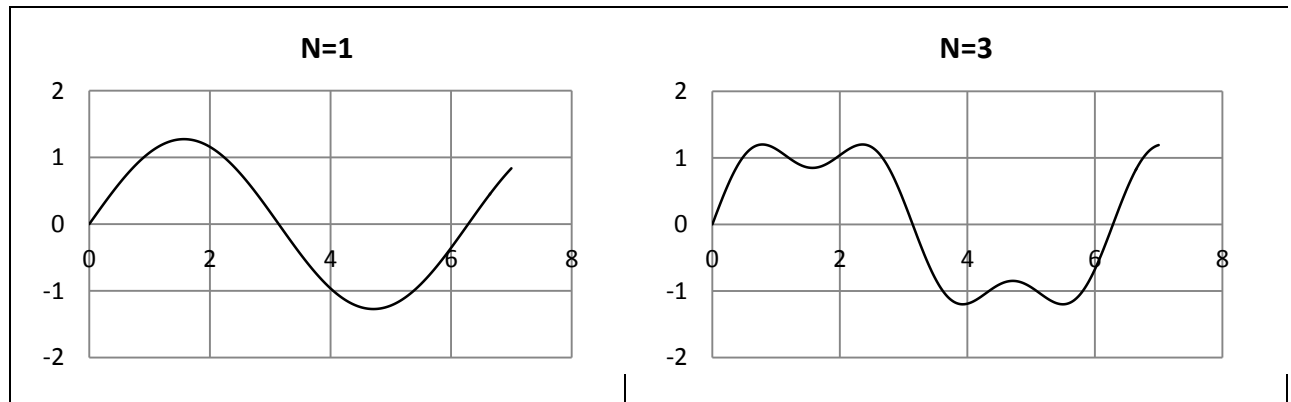
$$f(x) = 2 \left[H\left(\frac{x}{L}\right) - H\left(\frac{x}{L} - 1\right) \right] - 1 \quad \text{Eq.49}$$

where $H(x)$ is the Heaviside step function. Since $f(x) = f(2L - x)$, the function is odd, so $a_0 = a_n = 0$, and

$$\begin{aligned} b_n &= \frac{1}{L} \int_0^{2L} f(x) \sin\left(\frac{n\pi x}{L}\right) dx = \frac{4}{n\pi} \sin^2\left(\frac{1}{2} n\pi\right) \\ &= \frac{2}{n\pi} [1 - (-1)^n] \end{aligned} \quad \text{Eq.50}$$

The Fourier series is therefore

$$f(x) = \frac{4}{\pi} \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n} \sin\left(\frac{n\pi x}{L}\right) \quad \text{Eq.51}$$



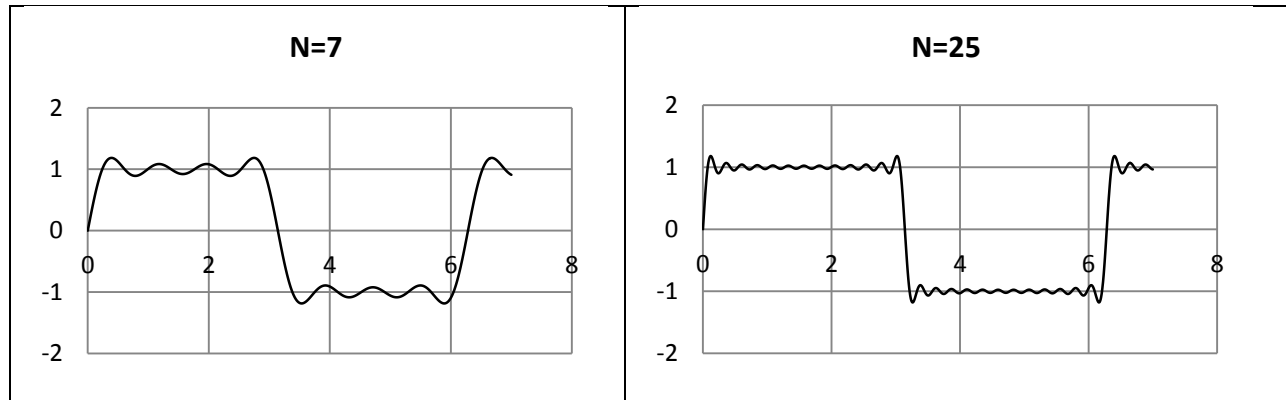


Figure 5 Fourier series expansion of a periodic square wave ($L=1$). The number of terms in the series varies from one, three, to seven and 25.

2.3.1.2 Introduction to Fast Fourier Transforms (FFT)

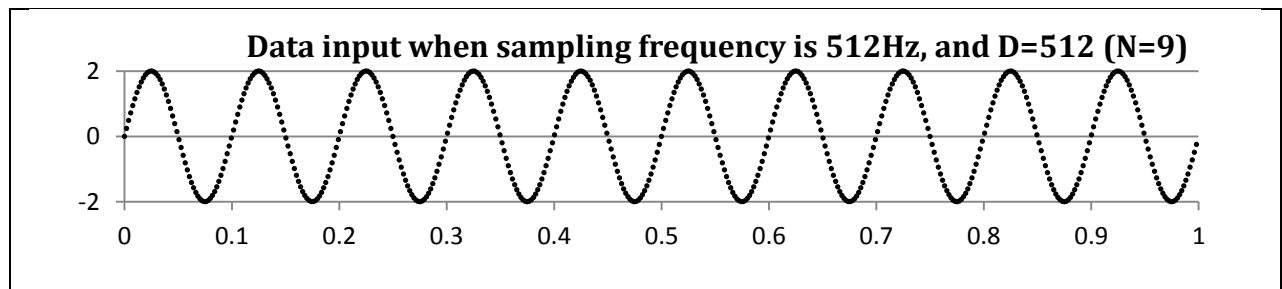
Fast Fourier transformation (FFT) is a technique used to rapidly convert data from time domain to frequency domain. It decomposes a sequence of values into components of different frequencies. The input to a FFT consists of a series of 2^n data points sampled in time domain at a constant sampling frequency (equally spaced intervals). The output consists of a series of $2^n - 1$ data points in frequency domain showing the contribution of each frequency to the overall signal. The resolution of the FFT is given by

$$\text{Resolution} = \frac{f_{\text{sampling}}}{D} \quad \text{Eq.52}$$

Sampling frequency f_{sampling} is determined by dividing the number of data points $D = 2^N$ by the time interval of sampling t :

$$f_{\text{sampling}} = \frac{D}{t} \quad \text{Eq.53}$$

Higher the sampling frequency, higher the accuracy of the FFT. Below are the FFT analysis of the function $f(x) = 2\sin(20\pi x)$, over the range of $[0, 1]$ second. The function has a frequency of 10 Hz. The input data and FFT analysis results are listed in Figure 26.



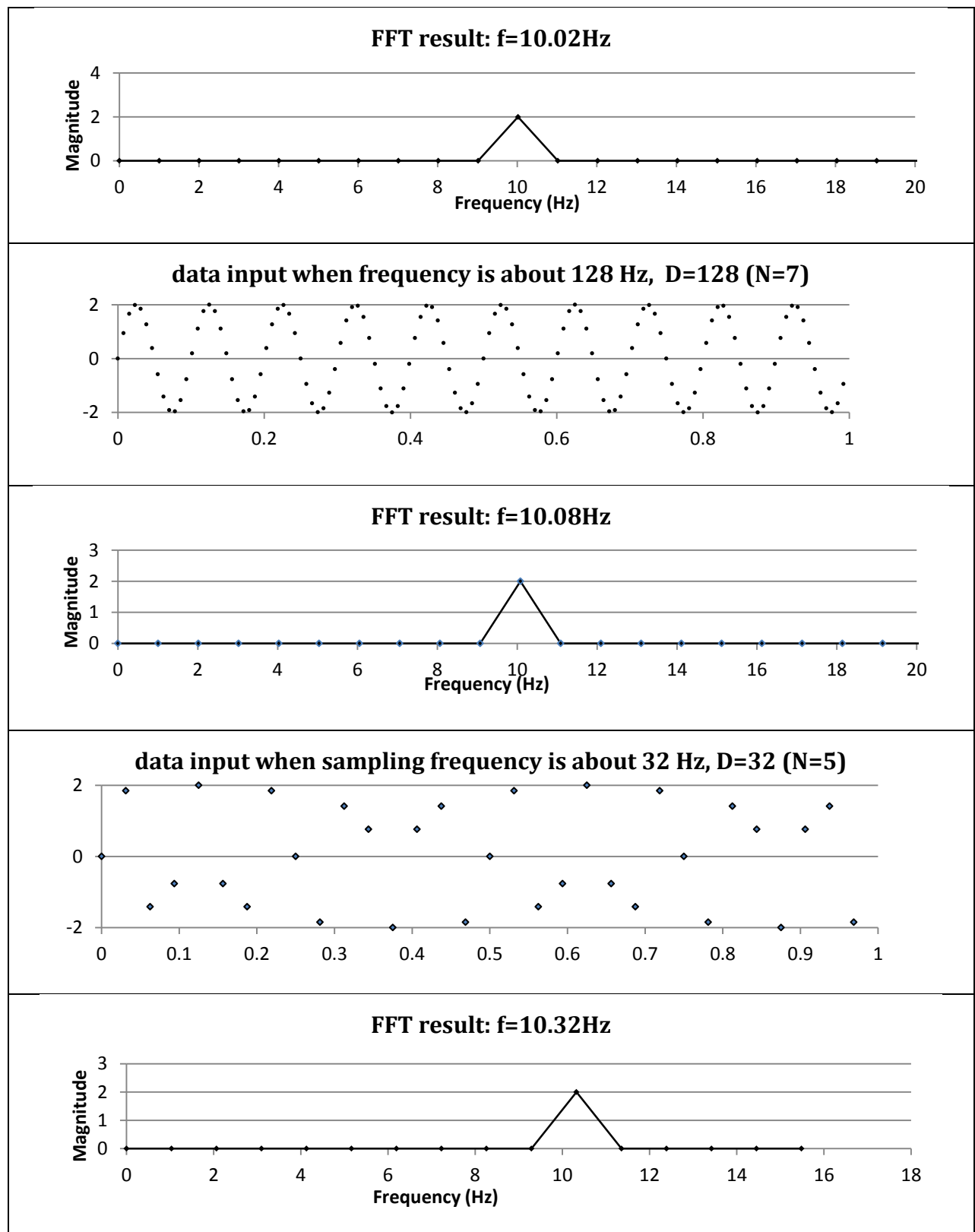


Figure 26 Data input to Fourier analysis and results

The result of FFT includes a real and an imaginary component. The magnitude (or power) and phase of the FFT data is computed by

$$\text{Magnitude} = \sqrt{\text{real}^2 + \text{imaginary}^2} \quad \text{Eq.54}$$

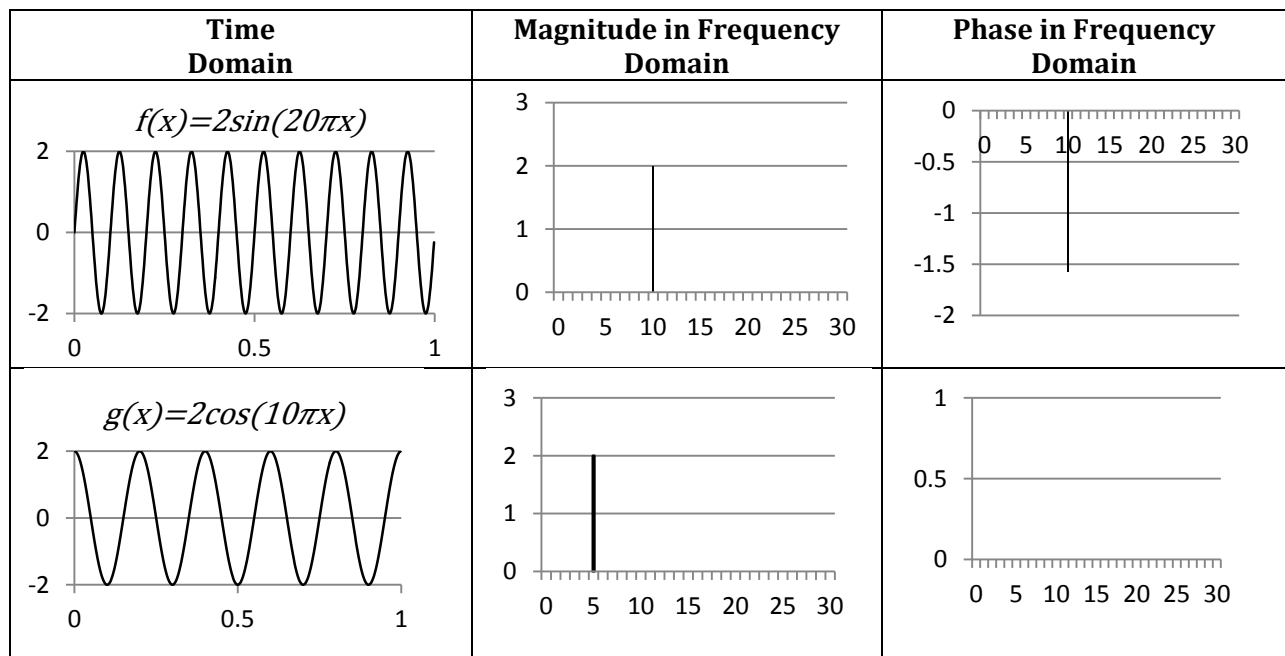
$$\text{Phase} = \tan^{-1}\left(\frac{\text{imaginary}}{\text{real}}\right) \quad \text{Eq.55}$$

For example, at 10Hz, the magnitude of the function $f(x) = 2\sin(20\pi x)$ has magnitude of 2, and a phase of -0.5π or -90° ; while the function $g(x) = 2\cos(10\pi x)$ has a magnitude of 2 and a phase of 0 at 5Hz.

2.3.1.4 Properties of Fourier Transforms

The Fourier transform is linear. It possesses the properties of homogeneity and additivity. That is, scaling in one domain corresponds to scaling in another domain, and addition in one domain correspond to addition in another domain.

Figure 27 shows scaling and addition of $f(x)$ and $g(x)$ mentioned in previous paragraph. We can clearly see that scaling the input in time domain results in same scaling in magnitude, but has no effect in phase. And addition of inputs in time domain correspond to a combination of magnitude and phase of the two inputs' frequency domain.



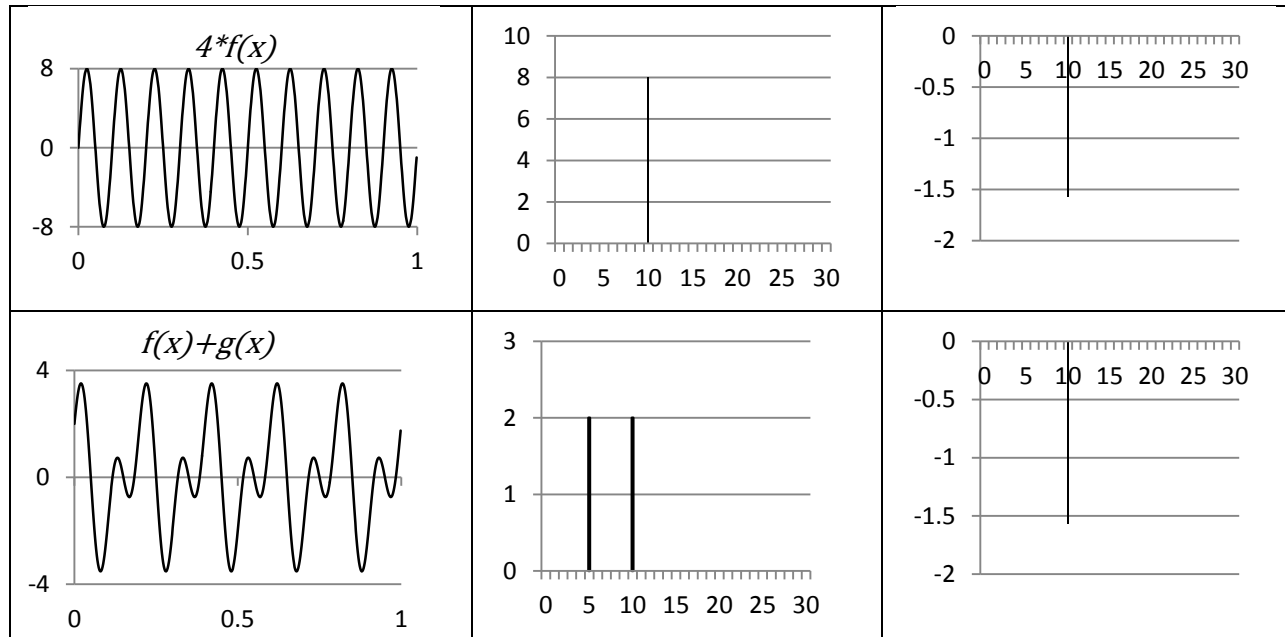


Figure 27 properties of Fourier Transformation

This additivity can be understood in terms of how sinusoids behave. Consider adding two sinusoids with the same frequency but different amplitudes) and phases. If the two phases happen to be the same, the amplitudes will add when the sinusoids are added. If the two phases happen to be exactly opposite, the amplitudes will subtract when the sinusoids are added. When sinusoids (or spectra) are in polar form, they cannot be added by simply adding the magnitudes and phases.

In spite of being linear, the Fourier transform is not shift invariant. In other words, a shift in the time domain does not correspond to a shift in the frequency domain. Instead, a shift in the time domain corresponds to changing the slope of the phase.

2.3.1.5 Examples of Fourier Transforms

Recall the periodic sawtooth function used in section 1, Fourier transforms can be used to find its frequencies. In Figure 28, the first “peak” in positive frequency domain indicates 0.5 Hz as the function’s first frequency. Note the offset of the function results in a peak of magnitude at 0 Hz, and the time shift in the function causes the shift in slope of phase. Compare the result after removing the offset and time shift (shown in Figure 29) with the original result.

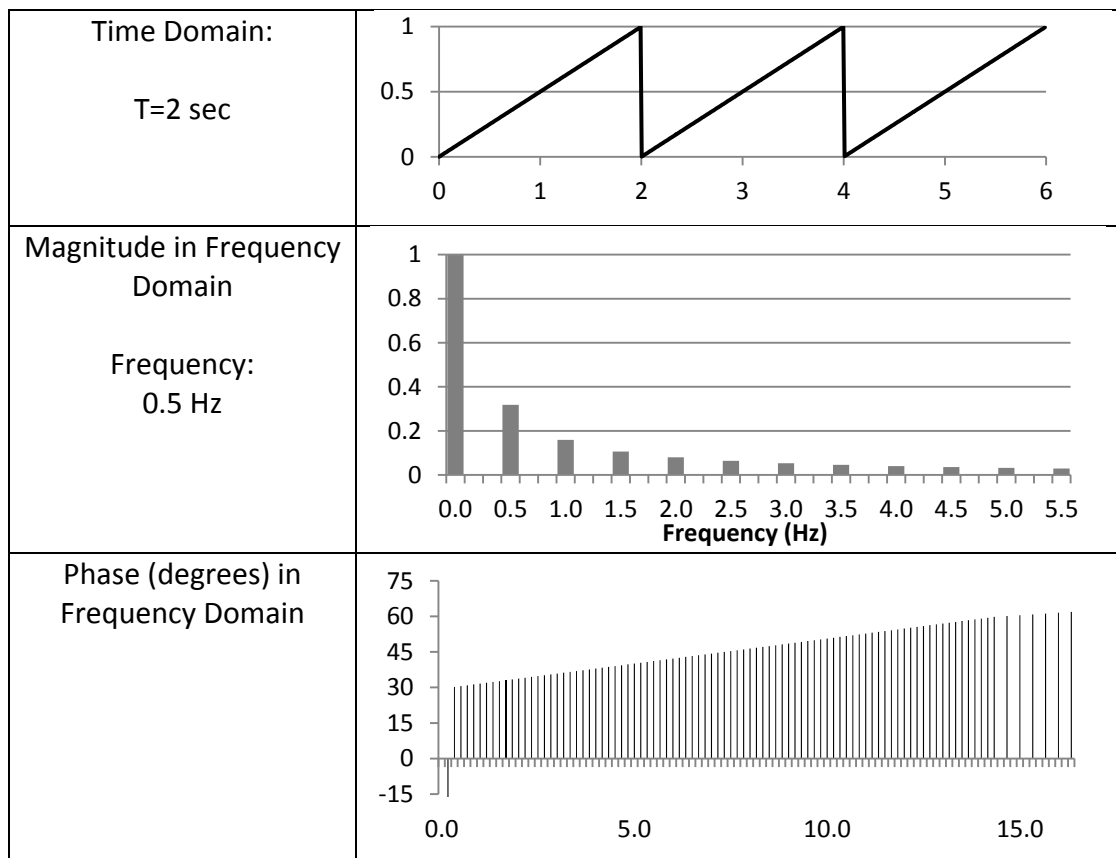
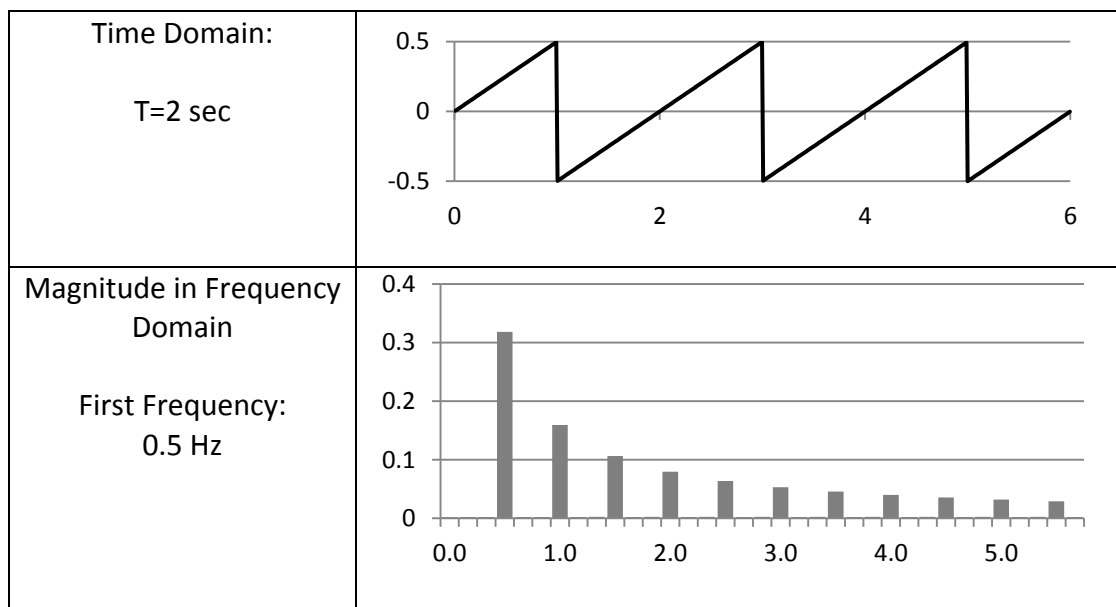


Figure 28 Fourier Transform of Periodic Sawtooth Function



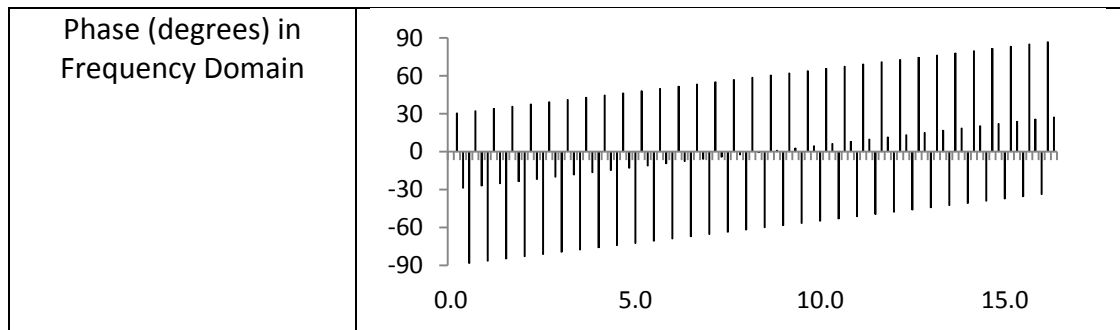


Figure 29 Fourier transformation of periodic sawtooth function without offset and time shift.

Applying Fourier transform to the periodic square wave function used in section 1 yield results in Figure 30. Comparing the results in Figure 31 with Figure 30, we can see that a time shift leads to a shift in phase, but have no impact on magnitude.

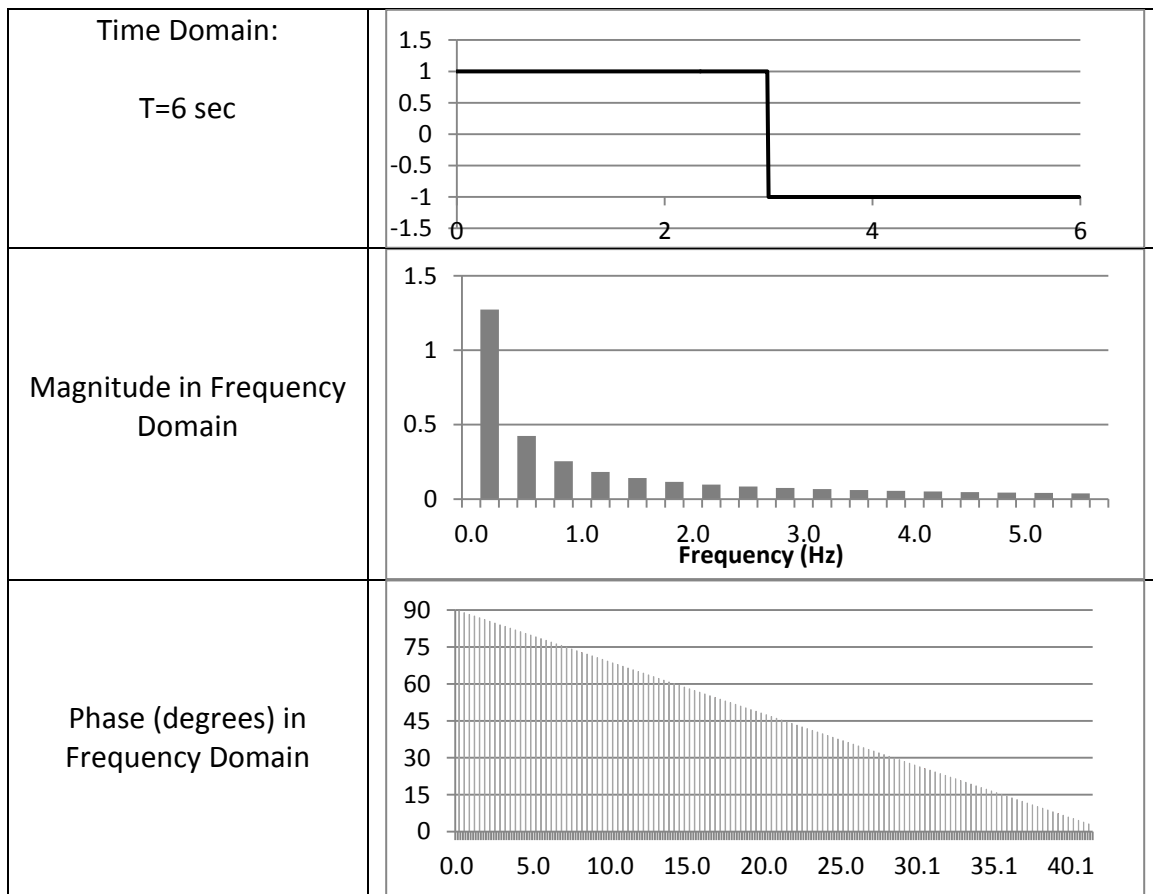


Figure 30 Fourier transform result of periodic square wave.

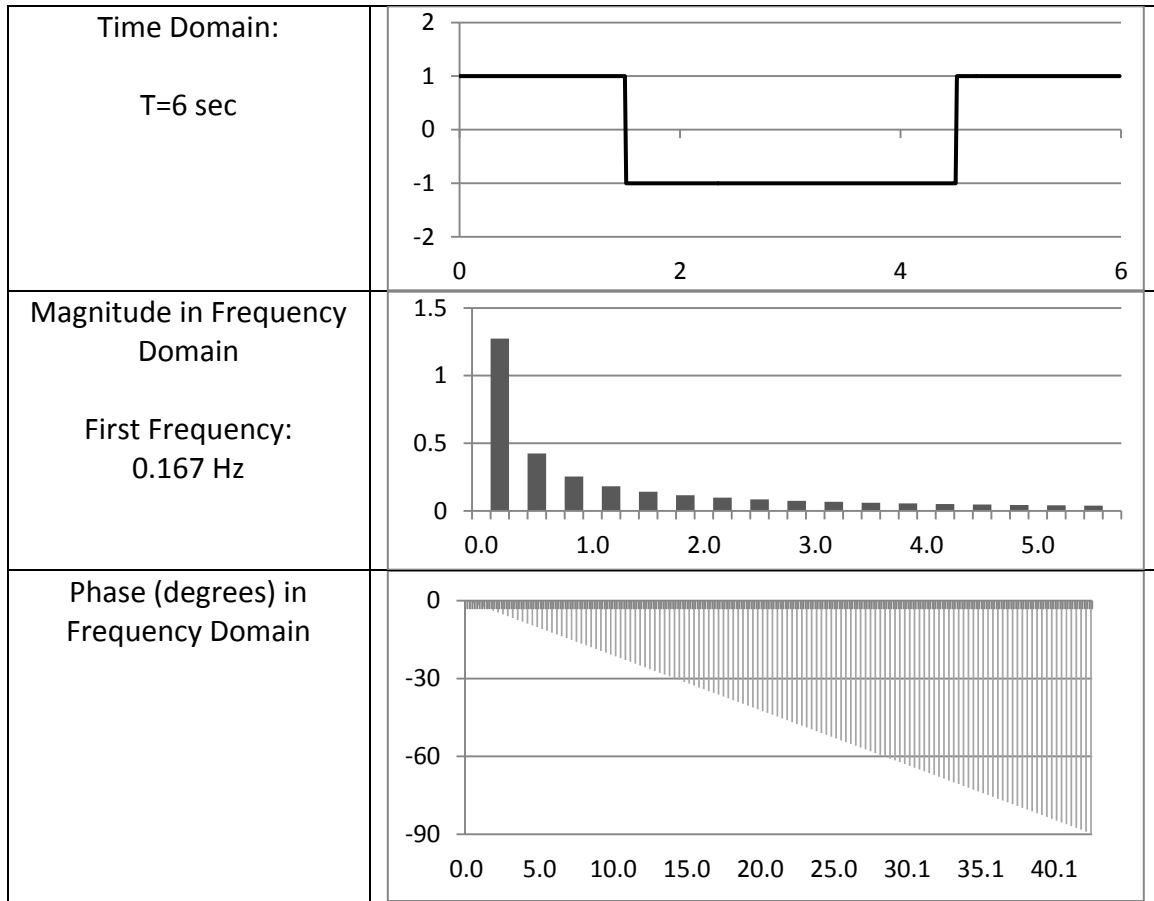


Figure 31 Fourier transform result of periodic square wave with time shift.

2.3.2 Determining Damping Factor: Logarithmic Decrement

Logarithmic decrement, δ , is used to find the damping ratio of an underdamped system in the time domain. The logarithmic decrement is the natural log of the ratio of the amplitudes of any two successive peaks, as shown in eq.36.

$$\delta = \frac{1}{n} \ln \left(\frac{y_i}{y_{i+n}} \right)$$

where y_i is the amplitude of peak i (i is an integer counting each peak), n is the number of cycles being considered, δ is the log decrement. If there is an offset in y , the y_i amplitude must be defined relative to that offset.

The damping ratio is then found from the logarithmic decrement, as shown in eq.37.

$$\zeta = \frac{\delta}{\sqrt{(2\pi)^2 + \delta^2}}$$

2.3.3 Determining Vibration Amplitude, Velocity, and Acceleration

Eq.28 shows the relationship between the deflection at the free end of the beam and at any point on the beam. The distance between the free end and the point is denoted by y .

$$w_y(t) = \left[1 - \frac{3}{2} \left(\frac{y}{L} \right) + \frac{1}{2} \left(\frac{y}{L} \right)^3 \right] w_{max}(t)$$

Eq. 9 and Eq.14 addressed the derivations of strain and deflection of the beam at a point with distance x from the clamped end.

$$\begin{aligned} \varepsilon &= \frac{P(L-x)T}{2EI} = \frac{6P(L-x)}{EbT^2} \\ w &= \frac{Px^2(3L-x)}{6EI} = \frac{2Px^2(3L-x)}{EbT^3} \end{aligned}$$

Therefore, the expression for the deflection can be updated:

$$w = \frac{\varepsilon x^2(3L-x)}{3T(L-x)} \quad \text{Eq.56}$$

Since it is obvious that $L=x+y$ for any point chosen, we have the expression for the peak altitude in terms of strain, and the location of measured strain, length of the beam, and thickness of the beam:

$$A_{peak} = w_{max} = \frac{\varepsilon}{3T(L-x)} \quad \text{Eq.57}$$

And taking a derivation in regards of time gives the peak velocity of the tip:

$$v_{peak} = \frac{dw_{max}}{dt} = \frac{d\varepsilon/dt}{3T(L-x)} \quad \text{Eq.58}$$

And a second order derivative of the deflection gives the peak acceleration:

$$a_{peak} = \frac{d^2w_{max}}{dt^2} = \frac{d^2\varepsilon/dt^2}{3T(L-x)} \quad \text{Eq.59}$$

The root mean square (abbreviated RMS), is a statistical measure of the magnitude of a varying quantity. It is especially useful when variants are positive and negative, e.g., sinusoids, RMS is used in various fields.

The RMS value of a set of values (or a continuous-time waveform) is the square root of the arithmetic mean (average) of the squares of the original values (or the square of the function

that defines the continuous waveform). In the case of a set of n values $\{x_1, x_2, x_3, \dots, x_n\}$, the RMS is given by:

$$x_{RMS} = \sqrt{\frac{1}{n}(x_1^2 + x_2^2 + x_3^2 + \dots + x_n^2)} \quad \text{Eq.60}$$

The RMS of a sine wave function $y = k \sin(2\pi ft)$ is given by:

$$y_{RMS} = \frac{k}{\sqrt{2}} \quad \text{Eq.61}$$

The RMS value of the vibration altitude, velocity and acceleration can be calculated by Eq.61 with the peak values provided by Eq.57, Eq.58 and Eq.59.

2.3.4 Determining the Elastic Modulus

Recall the expression of natural frequency in rad/sec in eq.22:

$$\omega_n = \alpha_n^2 \sqrt{\frac{EI}{mL^4}} = \alpha_n^2 \sqrt{\frac{EI}{\rho AL^4}} = \alpha_n^2 \sqrt{\frac{E(\frac{bT^3}{12})}{\rho(bT)L^4}} = \frac{\alpha_n^2}{2\sqrt{3}} \sqrt{\frac{ET^2}{\rho L^4}}$$

Since we have $\alpha_1 = 1.8751$, the first frequency in rad/sec can be expressed as:

$$\omega_n = 1.015 \sqrt{\frac{ET^2}{\rho L^4}} \quad \text{Eq.62}$$

Therefore, the elastic modulus can be given by:

$$E = \frac{0.9707 \omega_n^2 \rho L^4}{T^2} \quad \text{Eq.63}$$

2.4 Basics of Strain Gages

2.4.1 Operating Principle and Application of Strain Gages

Strain-gauge sensor is one of the most commonly used means of load, weight, and force detection. Strain gauges are frequently used in mechanical engineering research and development to measure the stresses generated by machinery, and in Aircraft component testing to structural measure stress of members, linkages, and any other critical component of an airframe.

A strain gauge operates on the principle that the electrical resistance of a wire changes when the length of the wire varies. It is used for measuring deformations in solid bodies. The strain experienced by the

sensor is directly proportional to the change in resistance of the gauge used, as shown in Eq 7. When unstressed, usual strain gauge resistances range from 30 Ohms to 3 kOhms.

Eq.64

$$R = \rho \frac{L}{A}$$

An ideal strain gage is small in size and mass, low in cost, easily attached, and highly sensitive to strain but insensitive to ambient or process temperature variations. The ideal strain gauge would undergo change in resistance only because of the deformations of the surface to which the sensor is coupled. However, in real applications, there are many factors which influence detected resistance such as temperature, material properties, the adhesive that bonds the gage to the surface, and the stability of the metal.

The strain sensitivity, which is also known as the gage factor (GF) of the sensor, is given by:

Eq.65

$$F = \frac{dR/R}{\epsilon_x}$$

where R is the resistance of the gauge without deformation, dR is the change in resistance caused by strain, and ϵ_x is the strain to be measured. Therefore, the strain can be expressed as:

Eq.66

$$\epsilon_x = \frac{1}{F} \frac{dR}{R}$$

2.4.2 Materials and Selection of Strain Gauges

Typical materials for strain gages include: constantan (copper-nickel alloy), nichrome v (nickel-chrome alloy), platinum alloys (usually tungsten), isoelastic (nickel-iron alloy), karma-type alloy wires (nickel-chrome alloy), foils, and semiconductor materials. The most popular alloys for strain gages are copper-nickel alloys and nickel-chromium alloys.

Temperature change can affect the internal structure of strain-sensing material, and also can amend properties of the material of the surface the strain gage is attached to. When there is a temperature change while a measurement is being made, the effects can cause large errors in data unless proper precautions are taken.

Each material has unique reaction to temperature change, as illustrated in figure below. Variation in expansion coefficients between the gage and base materials may cause dimensional changes in the sensor element. Therefore, it is a good practice to select strain gauge made of same type of material as the base structure.

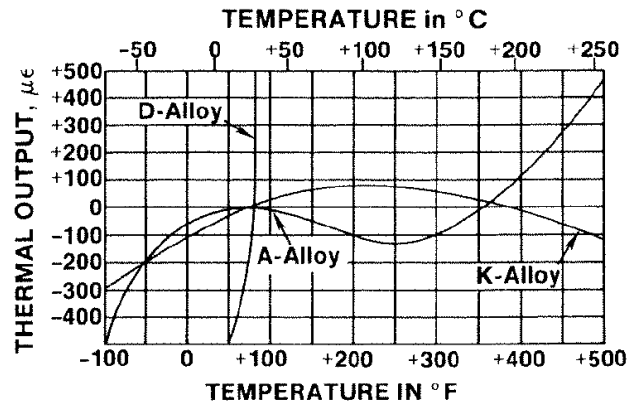


Figure 32 Temperature Effects on Thermal Output of Strain Gauges

Strain gauge's product name contains all critical information needed to select appropriate gauge. The meanings of each part of the name are shown in Figure 18 below. While Figure 19 shows key information of the type of strain gauge selected for this experiment.

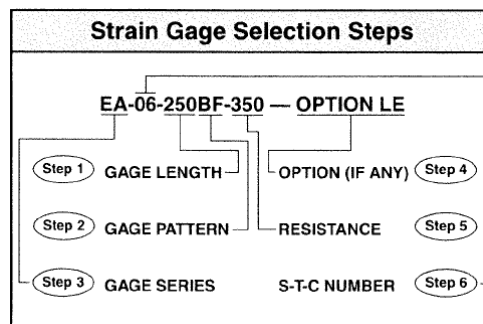


Figure 33 Strain Gage Selection Steps

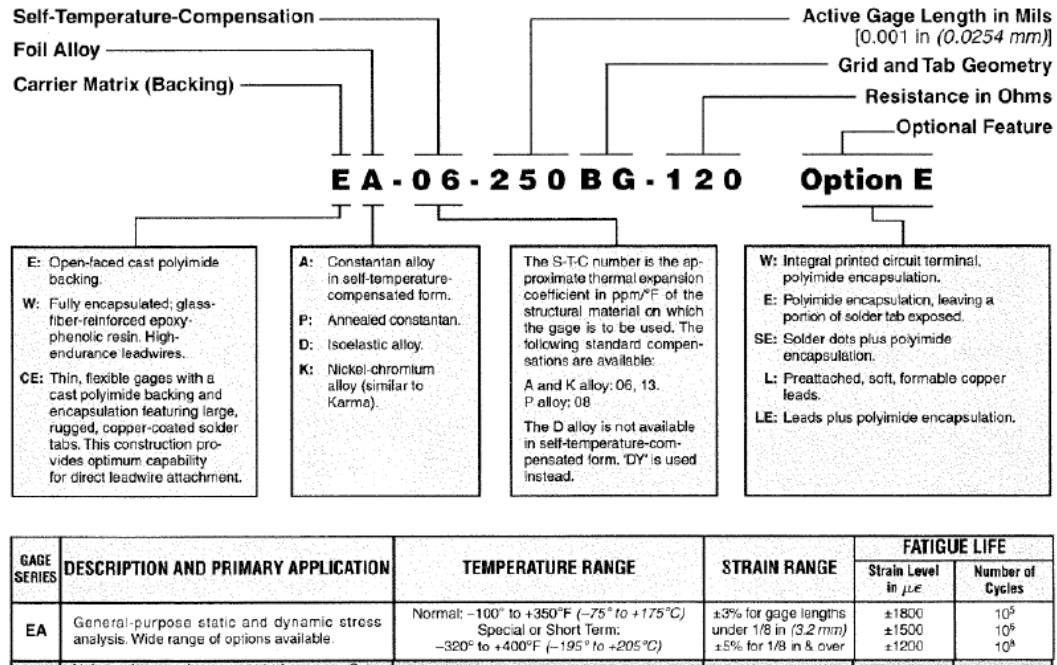


Figure 34 Crucial Information of Strain Gauge Selected

2.5 Basics of Wheatstone bridge

A Wheatstone bridge is an electrical circuit used to measure an unknown electrical resistance (from 1 Ω to 1M Ω) by balancing two legs of a bridge circuit, one leg of which includes the unknown component. A circuit diagram of Wheatstone bridge is shown in figure below, where the battery (symbol “E” serves as an excitation source, and the output is measured by a potentiometer “G”).

A “balanced” bridge is one with potential difference between B and D is equal to zero. Balance is sensed by closing switch S2 and measuring output current and voltage – to be near zero. Voltage drop across R2 is equal to voltage drop across R1, since voltage difference between B and D is equal to zero. Therefore,

$$R_x = \frac{R_1 R_3}{R_2} \quad \text{Eq.67}$$

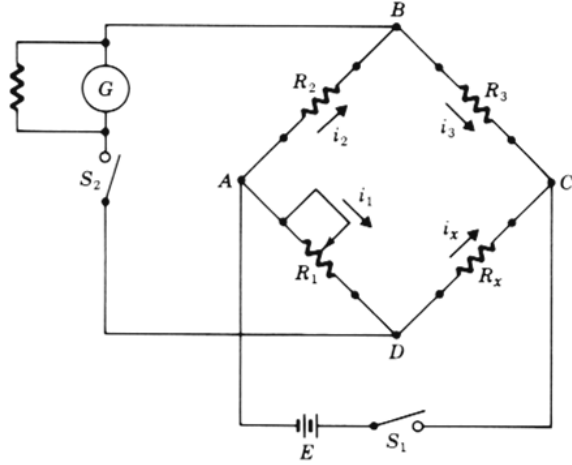


Figure 10 circuit diagram of Wheatstone bridge

When the bridge is unbalanced, equivalent resistance of the circuit is,

$$R = \frac{R_1 * R_4}{R_1 + R_4} + \frac{R_2 * R_3}{R_2 + R_3} \quad \text{Eq.68}$$

When the circuit is viewed as a circuit divider, the output voltage is,

$$E_g = \left(\frac{E}{R_1 + R_4} \right) * R_1 - \left(\frac{E}{R_2 + R_3} \right) * R_2 = E * \frac{R_1 R_3 - R_2 R_4}{(R_1 + R_4)(R_2 + R_3)} \quad \text{Eq.69}$$

When the resistance of R_4 changes by a small amount (ΔR_4), the new output voltage is,

$$E_g + \Delta E_g = E * \frac{R_1 R_3 - R_2(R_4 + \Delta R_4)}{(R_1 + R_4 + \Delta R_4)(R_2 + R_3)} = E \left(\frac{1 + \frac{\Delta R_4}{R_4} - \frac{R_3 R_1}{R_4 R_2}}{\left(1 + \frac{R_1}{R_4} + \frac{\Delta R_4}{R_4}\right) \left(1 + \frac{R_3}{R_2}\right)} \right) \quad \text{Eq.70}$$

If the bridge was originally balanced ($E_g = 0, R_1 = R_2 = R_3 = R_4$), then we have,

$$\Delta E_g = \frac{E * \frac{\Delta R_4}{R_4}}{4 + 2 \left(\frac{\Delta R_4}{R_4} \right)} \quad \text{Eq.71}$$

Since change in resistance is really small ($\Delta R_4 \ll 1$), the change in output voltage is,

$$\Delta E_g = \frac{E * \Delta R_4}{4 R_4}$$

or,

$$\Delta E_g = \frac{E * \Delta R}{4 R} \quad \text{Eq.72}$$

3. PROCEDURES

In order to determine the dynamic characteristics and elastic modulus of a vibrating cantilever beam, the procedures of this experiment include research relevant data, initial measurement of the beam, analytical estimations, hardware set-up, signal conditioning, testing with LabVIEW program, taking measurements, and data analysis.

The information acquired from research and part of the measurement process should also be used to produce uncertainty analysis and the contribution of each parameter to total uncertainty.

3.1 Preparations

3.1.1 Research of Relevant Data and Initial Measurements

In order to estimate the natural frequencies of the cantilever beam, the material's elastic modulus and the density need to be found from professional sources. Research online, or use a table in a textbook.

Measure the beam's length and thickness. With a pencil, mark the location to install the strain gauge on the beam at approximately 1 inch from the clamped end.

3.1.2 Undamped Natural Frequency

With the equations provided in Section 2, calculate theoretical undamped natural frequency of the beam. Make sure to use the actual effective length of the beam for calculation. The effective part of the beam is the “free vibrating” part between table and the free end.

3.1.3 Understand the Effect of Gain in the Signal Conditioner

Calculate the amplifier gain required to amplify the output of the Wheatstone bridge so that you get 1 mV/micro-strain.

Recall Gage factor,

$$F = \frac{dR/R}{\epsilon_x}$$

Measured strain can be expressed as

$$\epsilon_x = \frac{\Delta R}{FR_x} \quad \text{Eq.73}$$

Recall the expression of change in bridge output voltage caused by a small change in resistance ,

$$\Delta E_g = \frac{E \Delta R}{4R}$$

The relationship between measured strain and change in output can be found as,

$$\Delta E_g = \frac{F}{4} E \varepsilon_x \quad \text{Eq.74}$$

To achieve an output signal of 1mV per $\mu\varepsilon$, the gain (G) needs to satisfy:

$$\frac{G \Delta E_g}{\varepsilon_x} = \frac{1 \times 10^{-3} V}{1 \times 10^{-6}} = 1 \times 10^3 V$$

Therefore,

$$G = 1 \times 10^3 V \times \frac{4}{EF} \quad \text{Eq.75}$$

For this experiment, gage factor (F) is $2.095 \pm 0.5\%$.

3.1.4 Calculate the strain simulated by Shunt Resistors

Calculate the strain simulated by shunt resistors.

The connected shunt resistors are parallel to the gage, the equivalent resistance is:

$$R_{eq} = \frac{R_x R_{cal}}{R_x + R_{cal}} \quad \text{Eq.76}$$

Therefore,

$$\varepsilon_{eq} = \frac{\Delta R}{F * R_x} = \frac{R_{eq} - R_x}{F * R_x} = \frac{R_x}{F(R_x + R_{cal})} \quad \text{Eq.77}$$

Gage factor is $2.095 \pm 0.5\%$ for the gauge chosen for this experiment. Resistance without deformation is 120Ω .

3.2 Set-Up

3.2.1 Hardware Set-up

Clamp the beam to the edge of the lab bench. Place a metal plate between the clamp and the beam for noise reduction. Attach the strain gauge to the beam on the marked location.



Figure 35 Clamped Cantilever Beam with Strain Gauge Installed

Besides strain gauge and the beam, material needed for attaching the gauge to a surface include: sand paper, degreaser/alcohol, conditioner, neutralizer solutions, cotton balls & swabs, one-side sticky tape, adhesive, low-impedance strain gauge wire (about 15 “), and soldering material. The steps of are explained below.

- 17) Degreasing: wipe the surface with degreaser or alcohol to remove oil, grease, organic taminants and soluble chemical residues.
- 18) Surface abrading: sand the surface with sand paper, in order to remove loosely bonded adherents (scalp, rust, paint, coating, oxides, etc.) and develop a surface texture suitable for bonding.
- 19) Mark layout lines: mark the planned positions to attach strain gauges.
- 20) Apply neutralizer to the surface, alcohol works as well.

- 21) Mount on tape: secure strain gauge to the surface with tape, before applying adhesive. When mounting the gauge to the tape, make sure that the side of the gage with soldering terminals should be facing the tape, or “facing up” from the surface.

Carefully remove the strain gauge from its package with tweezers, make sure the strain gauge stay chemically clean. Attach one end of a 4-to-6 inch tape to the surface, carefully attach the strain gage to the tape with tweezers, then pick the gage up by lifting the tape at a shallow angle until the tape comes free with the gage and terminal attached. See figure below for illustration of this step.

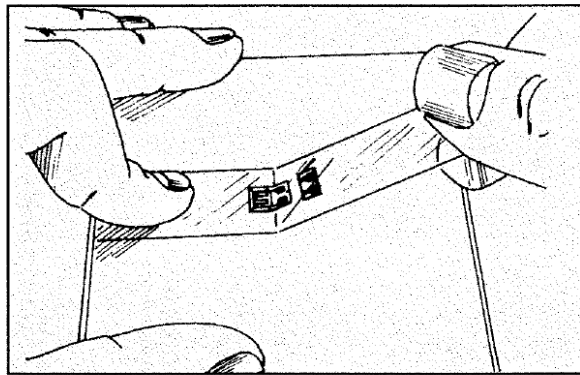


Figure 36 Mount the Strain Gauge on Tape

- 22) Position the tape: position the gauge/tape assembly so the gauge is over previously marked layout line. Gently apply the assembly onto surface. If the assembly is misaligned, lift the tape again at a shallow angle until the assembly is free from the surface. Reposition.
- 23) Lift tape: prior to applying adhesive, lift the end of tape opposite the solder tabs at a shallow angle, until the gauge and terminal is free from the surface. Tack the loose end of the tape under and press to the surface, so the gage lies flat with the bonding side exposed.

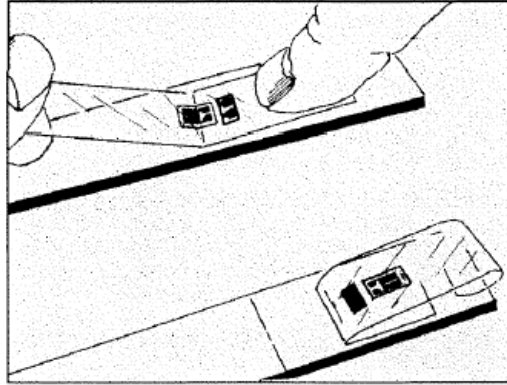


Figure 37 Lift tape

- 24) Apply adhesive and attach: apply a drop of adhesive to the gage's bonding side, attach the gauge and the surface by pressing on the tape for a minute. Wait two minutes before making a firm wiping stroke over the tape.
- 25) Remove the tape and clean the terminals with alcohol and a cotton swab.
- 26) Soldering and stress relief: mask the gage grid area with drafting tape before soldering. After soldering the wires to the terminals, tape the lead-in wires to the surface to prevent the wires from being accidentally pulled from the tabs.
- 27) Measure the base resistance of the unstrained strain gage after its proper mounting but before complete wiring. Check for surface contamination by measuring the isolation resistance between the gauge grid and the stressed force detector specimen by means of an ohmmeter, if the specimen is conductive. This should be done before connecting the lead wires to the instrumentation.
- 28) Strain gage should be connected to a Wheatstone bridge with quarter bridge set-up.
- 29) Connect the signal conditioner properly to provide power to the bridge and amplify the signal. For set-up procedures, refer to Document 2.
- 30) Connect the inputs from the signal conditioner to the NI DAQ device with a BNC cable, use channel AI0.

3.2.2 Construct the LabVIEW program

Refer to Document 3 for the tutorial to construct a basic VI program for this laboratory.

3.2.3 Verify the Set-up

Before starting the measurements, the strain gauge installations needs to be verified, the following steps should be followed:

- e. Run the VI program to monitor the readings.
- f. Check for irrelevant induced voltages in the circuit by reading the voltage when the power supply to the bridge is disconnected. Ensure that bridge output voltage readings for each strain-gage channel are practically zero.
- g. Connect the excitation power supply to the bridge and verify both the correct voltage level and its stability.
- h. Test out the strain gage bond by applying pressure to the gage. The reading should not be affected.
- i. Observe corresponding change in the time domain graph as the beam is gently bent.
- j. Take a weight provided by the lab and attach it to the beam, record a few seconds of voltage readings after the system stabilizes. Take an average of the stabilized data and calculate the corresponding measured strain. Calculate the theoretical strain at the point of the stain gauge and compare with the measurement result.

3.3 Taking Measurements

Set the sampling rate to over 1kHz. Pluck the beam a few times and record the data with provided program. Note that the program only records last group of data before clicking “stop” button to end the program. The length of the recorded data is the number of samples divided by sampling rate.

4. DATA ANALYSIS AND DISCUSSIONS

Determine the vibration amplitude, velocity, and acceleration in various units of measure; determine natural frequencies; measure and express damping characteristics as logarithmic decrement and percentage of critical damping; determine elastic modulus of a cantilever; compare measurements with analytical and/or computational models.

Conduct uncertainty analysis on the results. Assume 3% of uncertainty in strain measurements. Refer to provided sample uncertainty analysis.

Identify, in order of importance, percentage contribution of all uncertainties to the overall uncertainty in pressure characterizations and Poisson's ratio measurements.

*For optional activities during this laboratory, refer to Document 4.

Attachments

- Sample VI
- Sample Lab Report
- User Manual of Signal Conditioner Used in the Experiment

Document 1: Bending Stress and Strain in Cantilever Beam

Recall, the definition of normal strain is

$$\varepsilon = \Delta L / L \quad \text{Eq.1}$$

Using the line segments shown in Figure 1, the before and after length can be used to give

$$\varepsilon = \frac{\overline{A'B'} - \overline{AB}}{\overline{AB}} \quad \text{Eq.2}$$

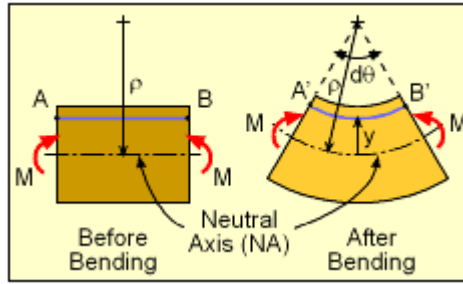


Figure 38 Bending of a Cantilever Beam

The line length on neutral axis remains same after bending. The length becomes shorter above the neutral axis (for positive moment) and longer below. The line AB and A'B' can be described using the radius of curvature, ρ , the differential angle, $d\theta$, and the distance from A'B' to the neutral axis, y . The y coordinate is assumed upward from the neutral axis, where there is no strain.

$$\overline{AB} = \rho d\theta \quad \text{Eq.3}$$

$$\overline{A'B'} = (\rho - y)d\theta \quad \text{Eq.4}$$

Therefore we have

$$\varepsilon = \frac{(\rho - y)d\theta - \rho d\theta}{\rho d\theta} = -\frac{y}{\rho} \quad \text{Eq5}$$

This relationship gives the bending strain at any location as a function of the beam curvature and the distance from the neutral axis.

The strain equation above can be converted to stress by using Hooke's law, $\sigma = E\varepsilon$, giving,

$$\sigma = Ey/\rho \quad \text{Eq.6}$$

This relationship between radius of curvature and the bending moment can be determined by summing the moment due to the normal stresses on an arbitrary beam cross section and equating it to the applied internal moment. This is the same as applying the moment equilibrium equation about the neutral axis (NA).

$$\sum M_{NA} = 0 \quad \text{Eq.7}$$

$$\int y(-dF) = M = - \int y\sigma dA \quad \text{Eq.8}$$

Combining Eq.7 and Eq.8 gives

$$\frac{E}{\rho} \int y^2 dA = M \quad \text{Eq.9}$$

Note that the integral is the area moment of inertia, I , or the second moment of the area. Using the area moment of inertia gives

$$\frac{EI}{\rho} = M \quad \text{Eq.10}$$

Eq. 10 can be used again to eliminate ρ , giving,

$$M = \frac{EI}{-\frac{Ey}{\sigma}} = -\frac{\sigma I}{y} \quad \text{Eq.11}$$

Rearranging gives,

$$\sigma = -\frac{My}{I} \quad \text{Eq.12}$$

This equation gives the bending normal stress, and is also commonly called the flexure formula. The y term is the distance from the neutral axis (up is positive). The I term is the moment of inertia about the neutral axis.

Document 2: Set-up Procedure for the Signal Conditioner (Tacuna)

d. Connection

Connect the wires as indicated in Figure 3.

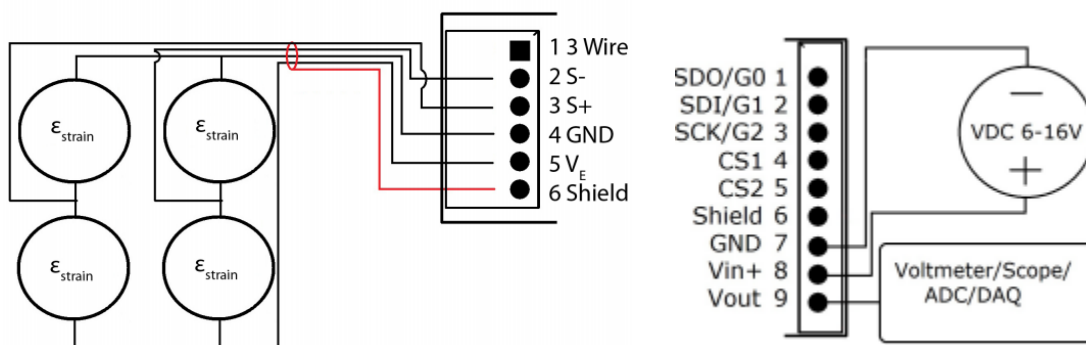


Figure 1 Connections for Tacuna Systems Strain Gauge or Load Cell Amplifier/Conditioner Interface Manual

e. Gain Setting

To get a gain of 220, make sure the switches (location shown in Figure 4) are set as indicated in Table 7.

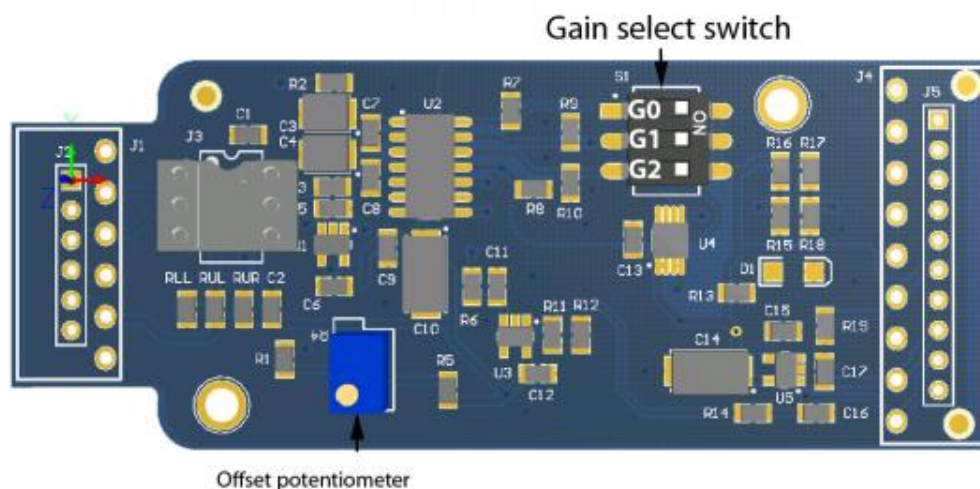


Figure 2 Location of Gain select switch and offset potentiometer

G0	G1	G2
ON	OFF	OFF

Table 10 Switch settings for Tacuna for 220 Gain

f. Bridge Balance

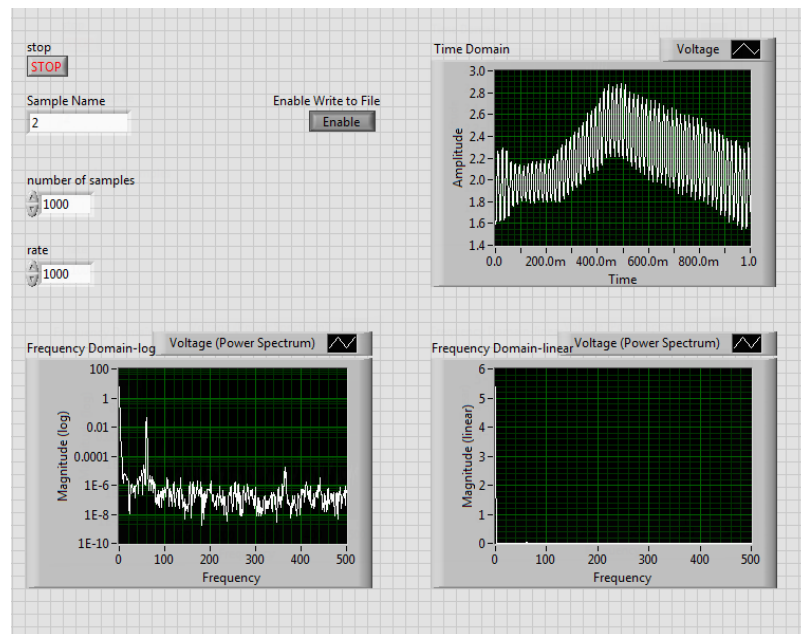
Use the offset potentiometer to adjust the output voltage to 2.5V, which is half of the output range.

It is required to open the enclosure to adjust the gain switches but not the offset potentiometer. The wire connections are located outside of the enclosure.

Document 3: LabVIEW Construction Tutorial

This sample LabVIEW program for the Vibration Laboratory acquires the voltage input from connected NI DAQ device, performs spectral analysis of the input over a specified time period, then saves data in both time domain and frequency domain to separate .csv files in the same folder where the LabVIEW program is saved. Around 1kHz acquisition rate is used for the experiment. This is a basic program to complete the experiment; there are many other ways to write an advanced VI.

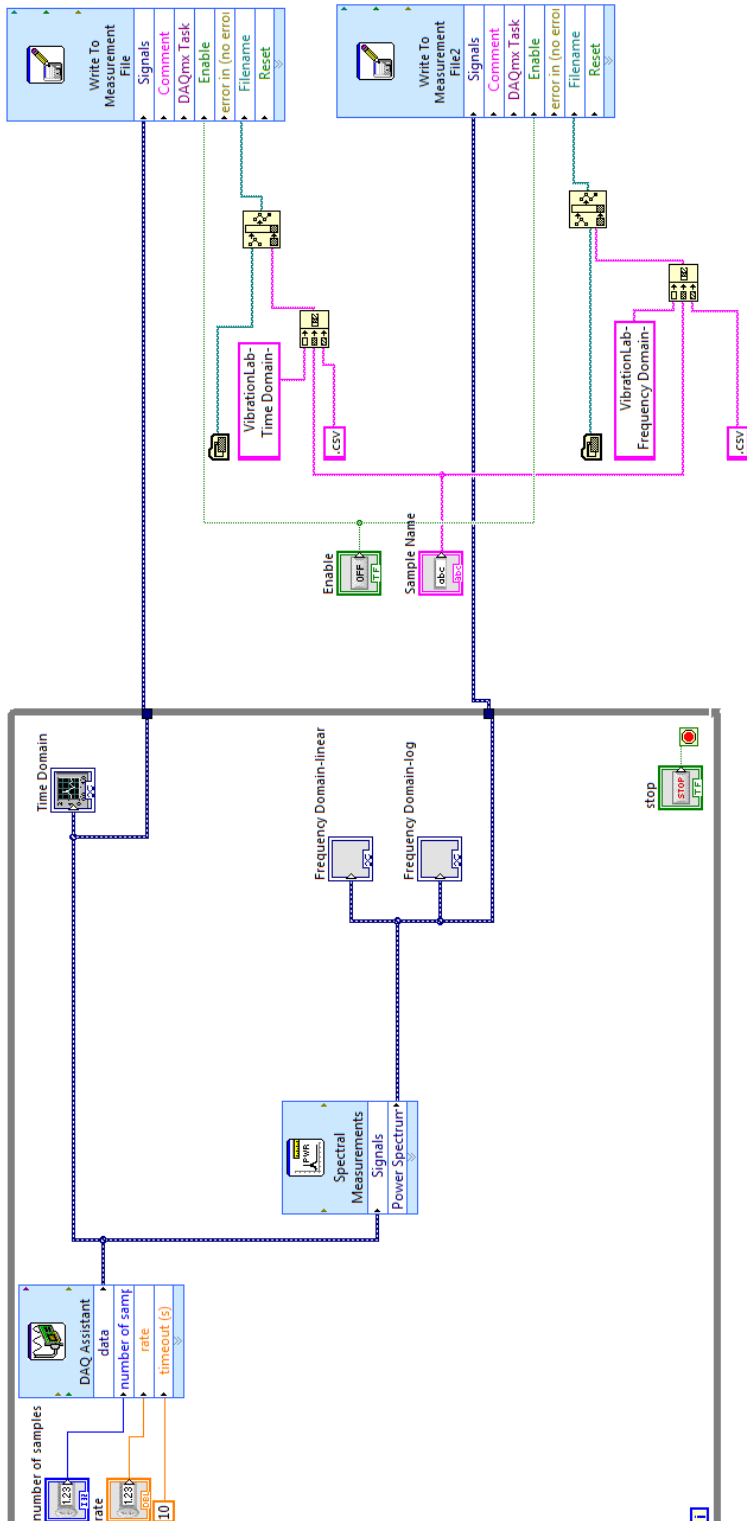
The front panel of the program is shown below. The block diagram is shown on page 2. This document walks through the steps of constructing this program.



Before opening LabVIEW program, make sure that the NI DAQ device is probably connected to the desktop and turned on.

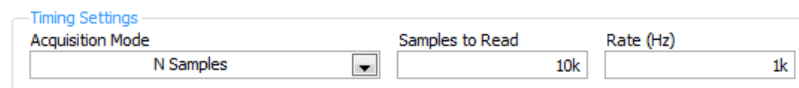
On *Tools Palette*, make sure that *Automatic Tools Selection* is enabled (the box/button on top of the palette). This setting automatically selects the appropriate pointer tools from the palette based on the mouse- over object.



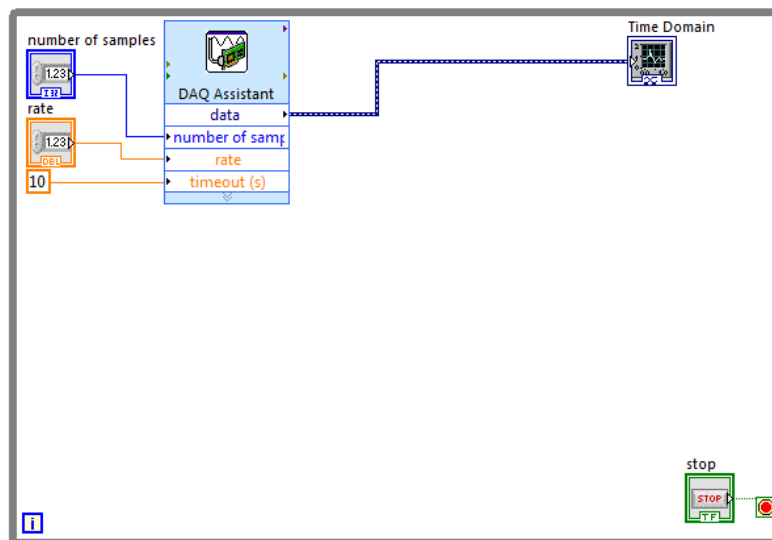


Add a *While Loop* and connect the (already created) *Stop Button* with the *Loop Condition* icon. (*Functions Palette* → *Programming* → *Structures* → *While Loop*). The modules can also be accessed by *Search* toolbox in *Function Palette*.

Add a *DAQ Assistant* in the *While Loop* and configure the subVI with the wizard. (*Functions Palette* → *Measurement I/O* → *NI DAQ mx* → *DAQ Assistant*). For the measurement type, select *Acquire Signals* → *Analog Input* → *Voltage*. For the physical channel, select the channel of incoming signal. Since channel AI0 of NI 6229 is connected to the input, select this specific channel. Next, configure the channel settings: *N Samples* for acquisition mode. Note that the DAQ box needs to be connected to the computer and turned on before starting of LabVIEW program. Save the work and restart the program if the module fails to initialize.

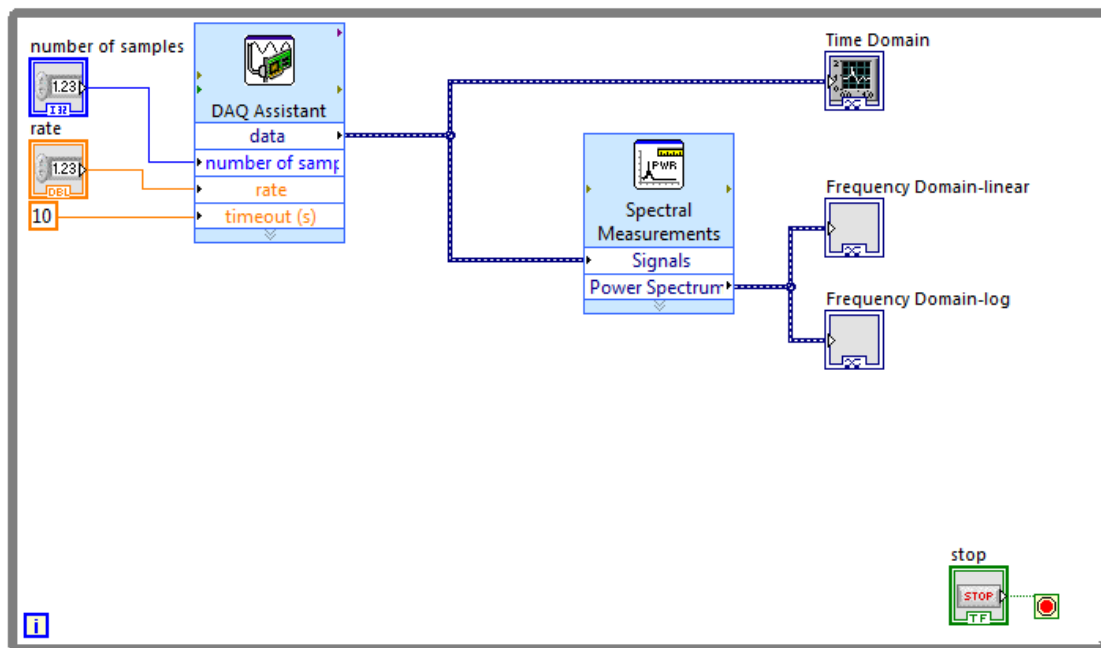
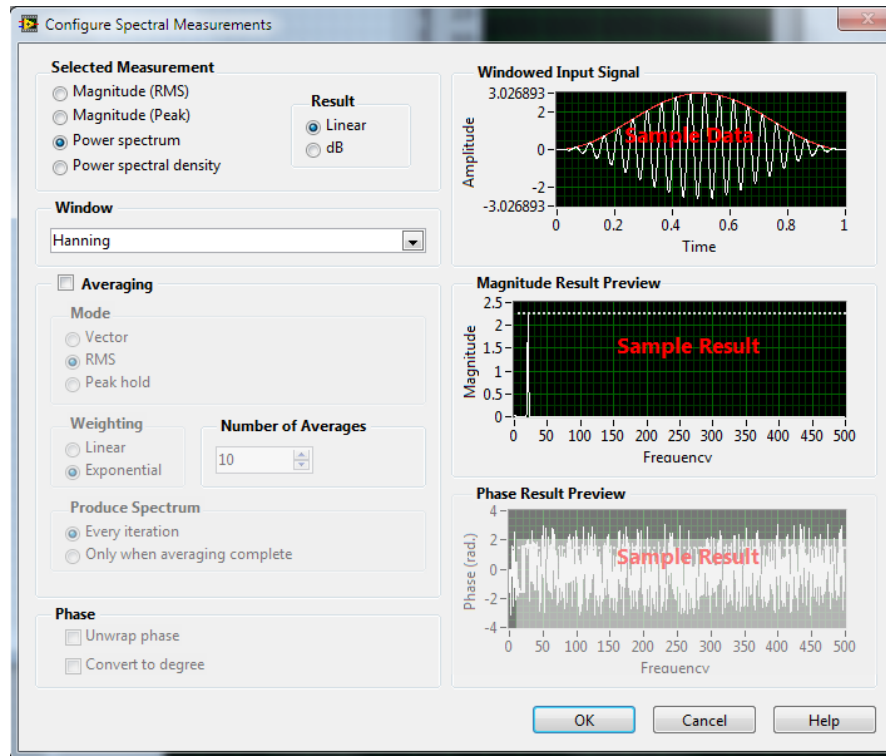


Drag down the downward arrow on the icon and create *Numeric Controls* for “number of samples” (samples to read) and “rate” (Rate Hz). The calculated timeout is the number of seconds for recorded data. Create a *Graphical Indicator* for data output of the *DAQ Assistant* and change the label of the graph into “Time Domain”.

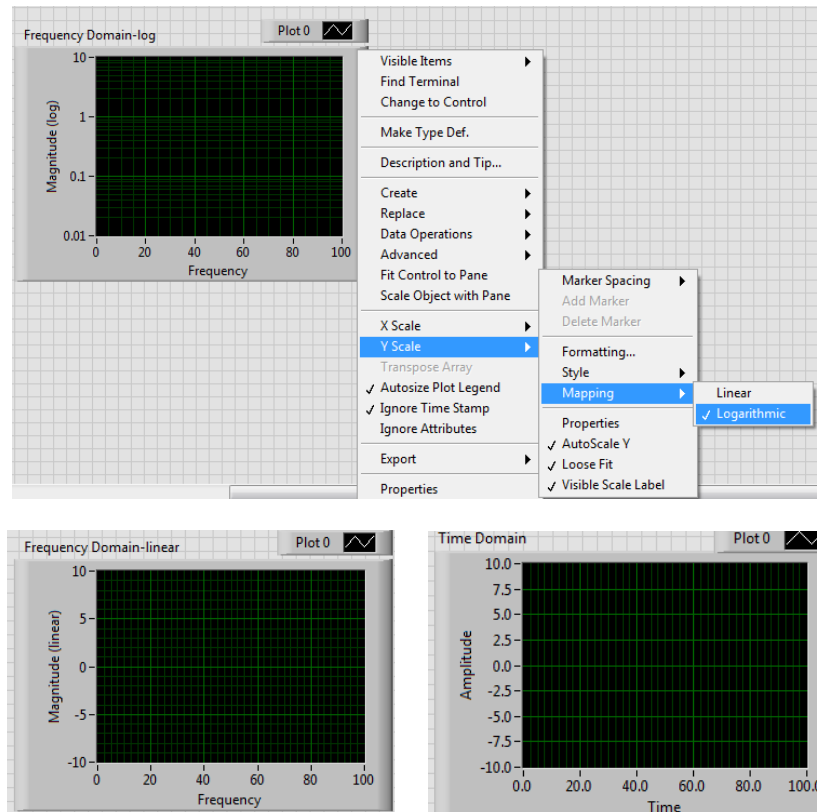


Create a *Spectral Measurement* for the data output of *DAQ Assistant* and create two *Graphical Indicators* for the power spectrum output of the function. In the configuration wizard,

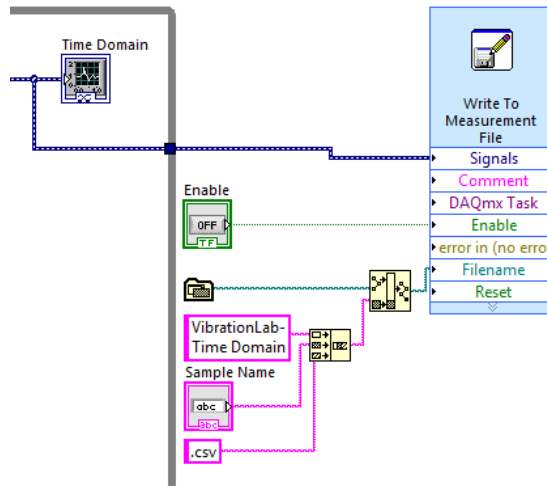
select “Power spectrum” as measurement. Change the labels of the *Graphical Indicators* into “Frequency Domain – Linear” and “Frequency Domain – Log”.



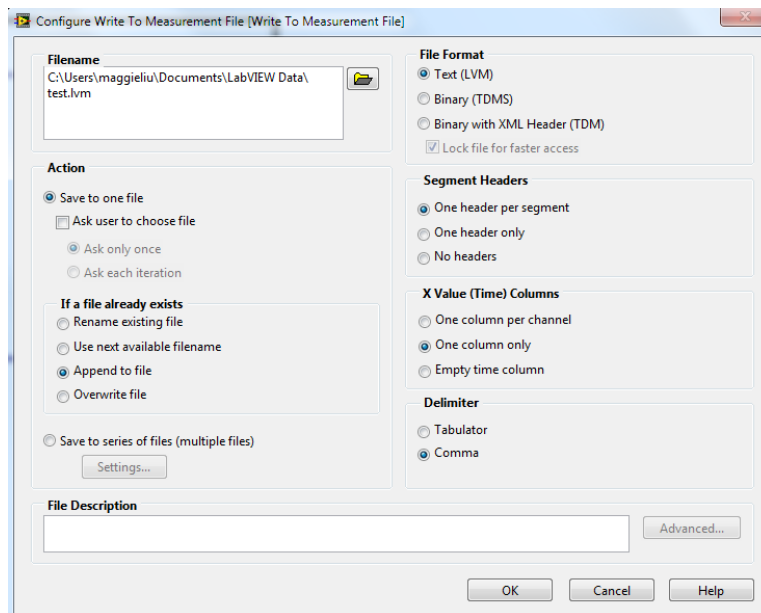
Go to *Front Panel* and configure the three *Waveform Graphs*. Replace the default axis labels with appropriate names (left clicking on the label texts enables editing). Make the mapping of Y axis on the Frequency Domain-Log graph “Logarithmic”; the menu is accessed by right clicking anywhere on the module.



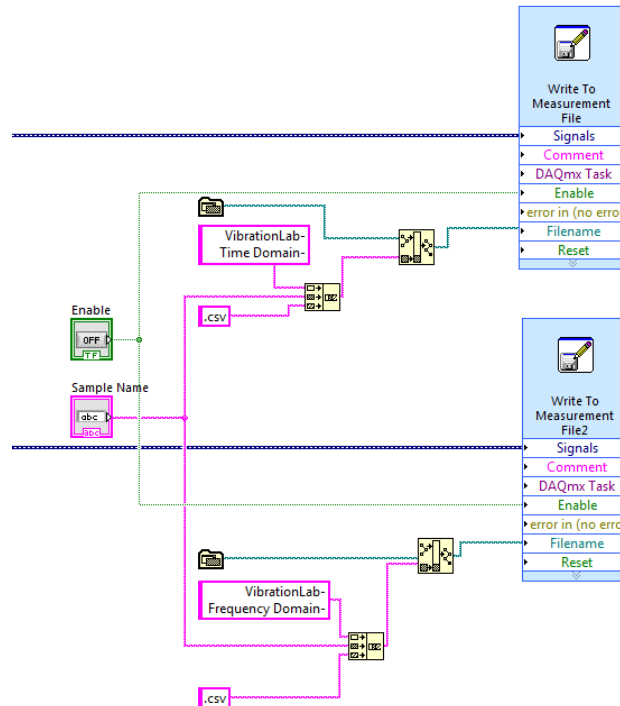
Create a *Write to Measurement File* module outside of the *While Loop*. Drag down the downward arrow to show the input and outputs of the module. Extend the *Dynamic Data* wire for the time domain data out of the *While Loop* and connect it to the signals input. Create a control for the “Enable” input, and rename the button “Enable Write to File”. The *Filename* can be constructed with *Build Path* function. It builds the file path with an *Application Directory* function, which points to the folder where the VI is saved, and a *Concatenated String* (Use *Concatenate String* function in *String Palette*) which consists of the lab name, the text “time domain data”, the user inputted sample name, and a “.csv” (comma separated values) as file extension, so that the data file can be opened with Microsoft Excel.



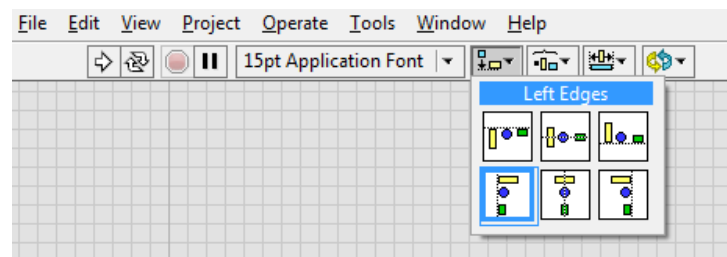
The *Write to Measurement File* should be configured as shown below. The filename in this wizard will be overwritten by the input; it should “save to one file”; the format should be text, with one header only or no headers; there should be only one time column; and the delimiter should be comma.



Create a second *Write to Measurement File* module for frequency domain data. The steps are the same as the other *Write to Measurement File*, so one could simple select all elements connected to the previous module and edit the elements later. The two modules should share the “Sample name” and Enable button. The filename of the second module should say frequency domain.

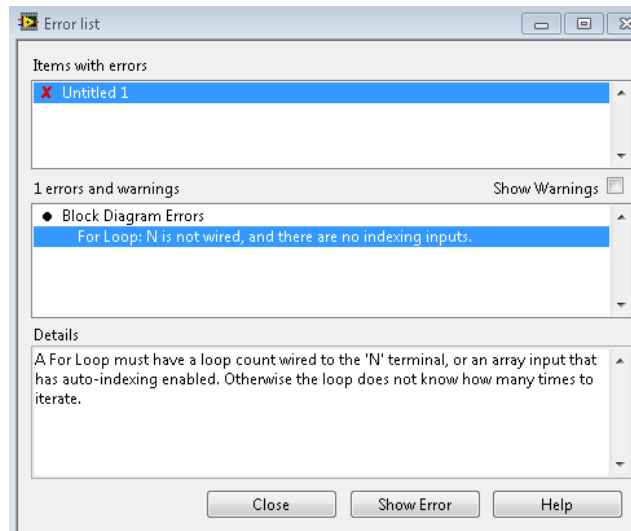


Rearrange the objects for a desirable layout. Drag the icon and drop them at appropriate locations. The objects can be arranged with the tools on the top tool bar, alignment, distribution and resizing tools can be used on selected objects.



Now we have completed constructing the VI. If there is any error in the program, the run button will appear “broken” as shown in the figure below. Click on the button to view the error list, the “details” should explain the error. Debug until all errors are resolved; use other debugging functions on the menu bar if needed.





When the run button appears as a rightward arrow, enter appropriate parameters on the *Front Panel*, connect a BNC cable to AI0 of the DAQ device with two idle clips (this will provide some varied voltage inputs), and test run the program. Use *Edit* ➔ *Make current values default* to save the entered parameters as default values. If there is no error interrupting the run, we can check the data file under the specified directory for satisfactory results. Trouble shoots until the program is ready for use.

Now the VI is ready for the Vibration Measurement Laboratory. Can you make it better?

Document 4: Optional Activities

1. Create a shared data file for the class; consolidate measured internal pressure from all the students. What is the average and standard deviation of the measured value? What are some of the possible causes of these variations?
2. Take two data recordings, one with the DAQ Assistants' input voltage range set to -10V to 10V, one with it set to -2V to 2V. Analyze the data and find out the resolution of each recording. Why are they different?
3. Read the user manual for the signal conditioner and change the gain setting. Compare the resolutions of strain readings under different gains.
4. A US nickel weighs 5 grams. Stack nickels on the further end of the beam after the completing the set-up of this experiment. Calculate measured strain when 1 to 10 nickels are on the beam. Compare the results with theoretical values. What are possible causes of deviations? Plot the results. If there is any nonlinearity, try to explain it.
5. Measure the elastic modulus in another way: apply various known weight onto the beam and plot the measured strains with calculated theoretical stresses. Modify the VI for this purpose if interested. Compare this measurement result with the result from vibration measurement and theoretical value.